Applied Compositional Thinking for Engineers (ACT4E)



Guest Lecture 1 - Dr. David Spivak

Questions & Answers

Q. A natural expectation for a "science of interoperability" would be to use software as a medium for doing this science. Will this necessitate more use of constructive mathematics, type theory and proof assistants? If so, are there any downsides to this approach?

Q. Is David suggesting a unification of science as sub-categories of physics?

David's answer to the question below "Q: Is category theory, through composability, inherently reductionist?" seems to imply an assumption of being able to reduce to "atoms".

I think he is saying that category theory is a language that can be shared between subfields of science (rather than physics being a fundamental category that all other science categories come from). E.g. if you can write something from quantum physics as a category of X, and you can write something from ecology as the same kind of category, then a quantum physicist would have intuition about this aspect of ecology without needing to study the field independently, similarly the ecologist would have an intuition about quantum physics without having to take physics courses.

Comment. The analogy of CT as a stem cell is great! It's not common origin that lets the resulting cells cohere, though, it's that the structure to cohere with is present in the stem cell.

Q. [Greg] David is clearly explaining how CT could be used to translate ideas between disciplines. However, following the analogy with stem-cells, is there a way in CT to combine/hybridize existing categories into a new category and understand quickier how the recombination is potentially fruitful in leading to new representations/results?

Q. Is an operad like an abstract syntax tree?

Q. Talking about unshared realities, has anyone built a category of historical accounts? [BP] What do you mean by a "historical account"?

[FJR] I mean when an historian writes an account of something, they have to select the sources they draw on to produce an account. Ideally you would like that account to stay the same in some sense if you were to add all possible sources, but that isn't possible in practice.

[BP] Ah, constructing a unified narrative from disparate sources; the narrative shifts when new perspectives are added. Instead of static stability, an asymptote is approached if the narrative is "complete". [FJR] I might say if the narrative is "accurate"

Q. [From Joshua Tan] Hi David :) there are these things called IPSCs: induced pluripotent stem cells, which are stem cells generated by inducing normal adult cells to "revert" to their stem cell forms. Often these are used to create stem-cell lines for specific patients, so scientists can study the specific mutations or diseases present in a patient, or to build extremely customized regenerative therapies. So... if CT is the stem cell and other fields of science are cells, curious what IPSCs would map to in your analogy? A specific, semi-automated trick for mapping any science into CT? Or a model in CT that "remembers" where the specific application it came from?

Could this be forgetful functors for example? (less specialisation from the structure of the category)

Q. So an operad is *not* a category? But it is *like* a category in that it has identities and composition with the same sorts of laws?

DS: Operads are more general than categories

Q: What is the key distinction that differentiates operads from monoidal categories? The definition for the morphisms seem to be just the hom-set Hom[tensorProd(X1,X2,X3,...), Y] while the objects could be contained within the objects of a monoidal category

Q. Sorts don't have identities? It doesn't make sense to talk about "two *different* boxes with no in/out ports"?

Q. Is there a representation theorem for operads? For example, is every operad representable as some kind of set of production rules?

Q: How does the operad of probabilities differ from the monad of probabilities?

Q: Silly question, but why is probability said to be a one sort Operad, where events are sorts... does this mean the probability can only have one event?

Q: How would context-*sensitive* grammars be framed in this operad-framework? Just curious, since I believe you can represent the phenomena of signal propagation with context-sensitive grammars, and that seems like a powerful mechanism to capture (ex - certain Lindenmayer-systems that model hormonal signaling).

Q: for organisers, do we have a link to the slides? GZ: We will ask David to make them available after the lecture **Q**: is there a relation between the meta-learning (a currently fashionable domain in machine learning) and category theory? Could category theory be helpful for meta-learning methods?

Q: I think I understand operads representationally. What's it like to calculate with/on them? GZ: You can let engineers that don't want to worry about the structure and its working principles just receive the result of the computation/run it

Q: Engineering is only concerned with computable systems. This creates a barrier between engineering's models and ethical concerns. David's motivation is ethical but are the assumptions behind this separation of hard science and the humanities not built in with this approach to interdisciplinarity? David's ethical concerns were about things like Fake News. Math isolates you from those questions.

You can write logic as mathematics, so anything you deduce in ethics with logic has this kind of structure. Also if you accept determinism in the brain (in that it can be model, or probabilities of time evolution can be predicted in order to account for quantum mechanics) then anything humans can come up with has some kind of mathematical underpinning you can work from.

AC: I disagree. Ethics (in a reduced meaning related to the field) is part of engineering. It appears in two meanings:

- 1) engineers should have ethical behavior
- 2) (more recent one) the system being built should produce ethical behavior.

Example for 1: IEEE code of ethics

Example for 2: Ethically aligned design . Working groups on ethically-aligned systems.

More generally, many types of engineers really are working on things that could kill people if designed badly; there are all sorts of ethical issues popping up.

(there are also engineers who work on things that are designed to kill people)

A recent paper we wrote exploring the ethical side of autonomous vehicle design: <u>https://psyarxiv.com/ypbve</u>

CS: Maybe it helps to reframe engineering as the art of making life better. Doesn't get any more ethical than that.

Comment on "social sciences don't use the same math": I think they do, but the relations are generally simpler and the range of relationships are much wider, so it's easier to rely on Wittgenstein style rule following to construct implicit language games than to try to formalize each different structure.

Q. What is the advantage of thinking in operads over thinking in algebras?

Q. If I think of operads as a set of types with primitive types and record types, how will that lead me astray?

Q: is the pixel matrix method computationally useful in practice? I.e. can it actually efficiently solve some otherwise difficult systems? (It's super cool)

Q: Is category theory, through composability, inherently reductionist?

What I understood from David's explanation: can work with whichever ontology you prefer (e.g. sociology). "You can keep diving down but you don't have to."

Q: The pixel array method looks like a discrete method applied to a continuous system, did you give a rigorous proof of the convergence when making the amount of pixels tending to infinity? Why does it only produce false positives?

Q: why has he multiplied the matrices to find the common solution of all three equations?

Q: what is the funny double arrow here?



It indicates a natural transformation (or a 2-morphism in general, a transformation between morphisms)

Q: [BP] Some graphs have directionality (causality) and other do not. How is this accounted for in the translation process?

Example: if you have two graphs, one with directionality, the other without. There is a "forgetful" functor from one to the other. Translation process is just about forgetting. If you were to translate back, you would need to include bidirectional arrows (you don't know what was the direction)

Q. Is there any link between Operads and Representation Theory?

Q. [ologs] What is the connection between operads and the ologs?

Background: (ontology logs, from Spivak, Kent "Ologs: a categorical framework for knowledge representation", with similar reasoning on diagrams serving as universal language of science).

Ologs = a way to attach English to categories; simplest possible way to get category theory out there, and explain what composition does. Could be worthwhile exploring attaching English to operads.

Q. Are there notable successes where category theory has led to concrete developments that transferred out of CT? I can think of UMAP as one.

Co-design! (the course will talk about this)

I found <u>https://mathoverflow.net/questions/19325/most-striking-applications-of-category-theory</u> and <u>https://mathoverflow.net/questions/83363/is-there-a-nice-application-of-category-theory-to-functional-complex-harmonic-an/83382</u> but no non-math answers.

Q. In all these abstractions we assume associativity. Is there a general theory without? It seems that in Physics this is becoming important. I mean something like a "category" without associativity. Like a Quandle rule for morphisms. Self-associativity for example.

DS: Example: Magmas; there is a category of magmas! (the magmas themselves don't have associativity)

Q: Is there a way to find optimal algebras for a given operad?

DS: for this one, you would need a notion of order, to say "this is better than that". There is a notion of "initial algebra".

- Perhaps this goes towards 'Is there a strategy to determine a suitable operad for a particular application?'

Q: Operads that have 2 inputs and 1 output are span's or sth more general? I am looking for theory for things that have 2 inputs and 1 output.

Q. David, do you have a reference about this idea of empowerment in learning?

DS: Polani (Daniel)

Q: Can you implement category theory in a graph database? Specifically constructs like Operads? Is there a data structure to rule them all? (or some primitives)

DS: (to second part) It is more about understanding each other's perspectives, not a method that rules them all

An example is Catlab; Can't say this is what you are looking for. <u>https://github.com/AlgebraicJulia/Catlab.jl</u>

Q: Can you say more about tooling for working with categories? By tooling I mean things like Categorical Query Language (CQL) or Labeled Property Graphs (LPG e.g. TinkerPop3, or Neo4j). It is my experience that DAGs and Labeled Property Graphs are inadequate for expressing categories (specifically with regard to functors, because arrows are not nodes). Some type of hypergraph is necessary. (I know that hypergraphs can be implemented with RDF or LPG but I find such implementations clumsy/busy.) I hope these issues will be addressed in TinkerPop4.