Applied Compositional Thinking for Engineers (ACT4E)

Session 4 - Combining

Questions & Answers

Q. Why does version 2 of the definition of bijectivity for functions include the reverse that g;f= id. That doesn’t seem necessary for functions. I assume you want it for categories, but it would be nice to motivate it.

NM: Because this is the general definition for morphisms, for functions you know your mapping is one-to-one, if you could allow functions that are one-to-many then you’d need the reverse to obtain the identity. But the key thing is that objects aren’t sets in general, for example let the morphism f:A->B indicate that A is at least as valuable as B. If you also have g: B->A then B is also at least as valuable as A and so the value of A and B is the same (this is assuming all homsets between any pair of objects either have one element or no elements). However in both these cases I haven’t assumed that the objects are sets (for example A could be a book on category theory and B could be a book on engineering)

As a counter example consider the sets {1}, {1,2}, you can define a function g:{1} -> {1,2} and a function f:{1,2} -> {1} then fg = id_{1}, but gf wouldn’t be an identity (it would map both 1 and 2 to either only 1 or only 2).

Yeah, sure. But then those two versions aren’t versions of the same thing! Yes, if you allow arbitrary relations, you need the second condition. It’s essentially saying the relation is a function.
NM: Does the idea that this is a more general thing make sense (and is one of the key things to understand about category theory)

Q: can we say that if A and B are isomorphic then if there exists a morphism A->C there must exist a B->C (for any C)? Can we say that Hom(A,C) and Hom(B,C) are isomophic ??

NM: I think so, let Hom(A,B) < {f} (using < as subset) where f is a isomorphism with inverse g then for all b_{j} \in Hom(B,C) then there exists a morphism b_{j} f \in Hom(A,C), similarly for every morphism a_{k} \in Hom(A,C) there is a morphism a_{k} g \in Hom(B,C). Furthermore we cannot have a_{k} g = a_{j} g unless a_{k} and a_{j} are the same since we would get a_{k} g f = a_{j} g f which implies a_{k} = a_{j}. This means that the mapping from composition of either f or g (on the valid side) is a one-to-one mapping and invertable.

GZ: I also think this is true. The converse, however, not. If Hom(A,C) and Hom(B,C) are isomorphic, it does not imply that A,B are.
NM: As a counter example to Hom(A,C) == Hom(B,C) implying A == B, just define your category so that there is only one morphism going from A->B but no morphisms going from B->A

Q: How about Hom(A,C) == Hom(B,C) and there exists f:A->B and g:B->A then f and g must be inverses of each other ???

NM: Only if Hom(A, A) = {Id_{A}} and Hom(B,B) = {Id_{B}}, for example let A be the integers, then gf may instead just be the add one to an integer operation. You could also have a non-invertable function so that gf could just set everything to zero.

Q: b is also in X and in Y. Is there an extra step to say the largest intersecting subset?

[EE]: So if I understood this correctly, the universality property is the reason why ‘intersection’ is a product, but ‘a common element’ isn’t.
GZ: The singleton {b} would “include” in {b,c}

Q. For the case of the product X intersect Y, what are the morphisms pi_1 and pi_2?
Inclusions. A way of seeing it is by considering the poset where order is given by inclusion. In that sense, this example is similar to this “Min” example in an ordered set.

NM: The intersection of sets is defined to be the categorical product of these two sets in this category. If we talk about a set that can be included in both X and Y, then it should be a subset of X intersect Y and so there is an inclusion into the categorical product if there is an inclusion into both X and Y.

OK, here’s what I think is so confusing (to me, at least, about this). When you show a particular category and you say that the objects are sets, that seems to suggest that we have morphisms that correspond to functions between the sets (as in the Set category). In that context, the cross product is a categorical product, and it makes sense because it retains the information of both elements as it were. But when you claim that the intersection of sets is a categorical product, even though the objects of the category are also sets, you have to say that this is true only in a category in which the ONLY morphisms are <= relations and composition is interpreted as relational composition---and no other morphisms are allowed. I suppose what this amounts to is that being a product is defined by the formula Jonathan gave, but the universal quantification in that formula should be not over all *possible* morphisms but only over the morphisms that actually exist in that particular category. So by restricting the set of morphisms enough, you can come up with a notion of product that is somehow not information-rich enough to represent both components. Is that right?

Q [from Zulip]. On page 21, shouldn't you have R5 instead of R7?
Actually, anything will do. Look at the definition on page 18: “For any P”. An example of P was just chosen here to be R7