## Applied Compositional Thinking for Engineers (ACT4E)



## Session 1b

## Questions \& Answers

Q:Question related to the Lego Associativity example: If one leaves the red brick, and composes the blue and white one, this does not seem to work?
GZ: Sure, the order matters!
Thank you

Q: I think drawing composition like this has some limitations, e.g. start with a,b,c,d, then compose $\mathrm{a}, \mathrm{b}$ to e,f, and $\mathrm{c}, \mathrm{d}$ to $\mathrm{g}, \mathrm{h}$, then if we want to compose $\mathrm{f}, \mathrm{g}$ and e,h, we get crossing lines... how would we deal with this?
Abcd
-------
Efgh
Now how to compose e and h ?

JL: I'm not $100 \%$ sure, but I think the rules with this kind of notation will allow you to turn this Abcd
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Efgh
----
into this

Abcd
-
Efgh
and then you can compose E and h (and f and g can just come along for the ride)
An answer to the "puzzle":
Associative but not commutative: Matrix multiplication
List of identity elements:
<N,+> -> Id is 0
$<R,+>$-> Id is 0
$<$ N, *> -> Id = 1
$<R$, *> -> Id = 1
$<\mathrm{N}$, max> -> Id is 0
$<N$, min> -> does not exist
<R, max> -> does not exist
$<R$, min> -> does not exist
$<\{x \operatorname{lin} N, x>=2021\},+>->$ does not exist
$<\{x$ lin $N, x>=2021\}$, *> -> does not exist
$<\{x \operatorname{lin} N, x>=2021\}, \max >$ id $=2021$
$<\{x \operatorname{lin} N, x>=2021\}$, min> -> does not exist

Q: If we consider a delay the linear systems composition is not linear anymore though (and thus it would not be a semigroup either), am I missing something?
Edit: ok the D matrix is missing, I see

## Which are groups: list of inverse elements

$<R,+, 0>=-x$
$<R$, *, $1>=$ no (unless we exclude $x=0$ )
$<\mathrm{N},+, 0>=$ no
$<\mathrm{N},{ }^{*}, 1>=$ no
$<\mathrm{N}$, max, $0>$ no

Q: Can you explain the monoid LTI system again please? What exactly is a neutral element? A:See explanation

Q: What is the formula for the matrices of a composition of linear dynamical systems?
A: Consider two systems

$$
\begin{aligned}
& x(k+1)=A x(k)+B u(k) \\
& y(k)=C x(k)+D u(k)
\end{aligned}
$$

$y(k)$ (the output of the first) is the input for the second system.
$z(k+1)=E z(k)+F y(k)$
$w(k)=G z(k)+H y(k)$,
$z(k+1)=E z(k)+F(C x(k)+D u(k))$
$=E z(k)+F C x(k)+F D u(k)$
$\mathrm{w}(\mathrm{k})=\mathrm{Gz}(\mathrm{k})+\mathrm{H}(\mathrm{Cx}(\mathrm{k})+\mathrm{Du}(\mathrm{k}))$
$=\mathrm{Gz}(\mathrm{k})+\mathrm{HCx}(\mathrm{k})+\mathrm{HDu}(\mathrm{k})$
You can write it as:

$$
\begin{aligned}
{\left[\begin{array}{l}
x_{k+1} \\
z_{k+1}
\end{array}\right] } & =\left[\begin{array}{cc}
A & 0 \\
F C & E
\end{array}\right]\left[\begin{array}{l}
x_{k} \\
z_{k}
\end{array}\right]+\left[\begin{array}{c}
B \\
F D
\end{array}\right] u_{k} \\
w_{k} & =\left[\begin{array}{ll}
H C & G
\end{array}\right]\left[\begin{array}{l}
x_{k} \\
z_{k}
\end{array}\right]+H D u_{k}
\end{aligned}
$$

Q: Could you please give more examples of applications of these concepts in practice? GZ:

- Electric circuits, resistors composition (in parallel or series)
- Mechanical springs composition

Q: Is there a place for donations / tips for your time?
GZ: Not for now, we are ETH employees. Fun artworks representing us would be appreciated
Q: when will the exercises be released?
GZ: More information will follow on Wednesday

