Applied Compositional Thinking for Engineers (ACT4E)



Session 1b

Questions & Answers

Q:Question related to the Lego Associativity example: If one leaves the red brick, and composes the blue and white one, this does not seem to work? GZ: Sure, the order matters! Thank you

Q: I think drawing composition like this has some limitations, e.g. start with a,b,c,d, then compose a,b to e,f, and c,d to g,h, then if we want to compose f,g and e,h, we get crossing lines... how would we deal with this?

Abcd

-----Efgh

_ · y

Now how to compose e and h?

JL: I'm not 100% sure, but I think the rules with this kind of notation will allow you to turn this A b c d

-----Efgh

into this

Abcd

Efgh

and then you can compose E and h (and f and g can just come along for the ride)

An answer to the "puzzle": Associative but not commutative: Matrix multiplication

List of identity elements: $\langle N, + \rangle \rightarrow Id \text{ is } 0$ $\langle R, + \rangle \rightarrow Id \text{ is } 0$ $\langle N, * \rangle \rightarrow Id = 1$ $\langle R, * \rangle \rightarrow Id = 1$ $\langle N, max \rangle \rightarrow Id \text{ is } 0$ $\langle N, min \rangle \rightarrow \text{ does not exist}$ $\langle R, max \rangle \rightarrow \text{ does not exist}$ $\langle R, min \rangle \rightarrow \text{ does not exist}$ $\langle R, min \rangle \rightarrow \text{ does not exist}$ $\langle X \text{ in } N, x \rangle = 2021 \}$, $+ \rangle \rightarrow \text{ does not exist}$ $\langle X \text{ in } N, x \rangle = 2021 \}$, $max \rangle \text{ id } = 2021$ $\langle X \text{ in } N, x \rangle = 2021 \}$, $min \rangle \rightarrow \rangle$ does not exist

Q: If we consider a delay the linear systems composition is not linear anymore though (and thus it would not be a semigroup either), am I missing something?

Edit: ok the D matrix is missing, I see

Which are groups: list of inverse elements

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<R, +, 0>= -x
<R, *, 1>= no (unless we exclude x=0)
<N, +, 0>= no
<N, *, 1>= no
<N, max,0> no
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Q: Can you explain the monoid LTI system again please? What exactly is a neutral element? A:See explanation

Q: What is the formula for the matrices of a composition of linear dynamical systems? A: Consider two systems

 $\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$

y(k) (the output of the first) is the input for the second system.

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z(k+1)= Ez(k)+Fy(k)

w(k)=Gz(k)+Hy(k),

z(k+1)=Ez(k) + F(Cx(k)+Du(k))

= Ez(k) + FCx(k) + FDu(k)

w(k)=Gz(k) + H(Cx(k)+Du(k))

= Gz(k) + HCx(k) + HDu(k)
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You can write it as:

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ FC & E \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} B \\ FD \end{bmatrix} u_k$$
$$w_k = \begin{bmatrix} HC & G \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + HDu_k$$

Q: Could you please give more examples of applications of these concepts in practice? GZ:

- Electric circuits, resistors composition (in parallel or series)
- Mechanical springs composition

Q: Is there a place for donations / tips for your time?

GZ: Not for now, we are ETH employees. Fun artworks representing us would be appreciated

Q: when will the exercises be released?

GZ: More information will follow on Wednesday