Applied Compositional Thinking for Engineers (ACT4E)

Session 2b

Questions & Answers

Q: In the sprouts/state space example it appears that the operation, T, is a unary rather than binary operation. Is this true and if so, how does this fit in with the definition of a semigroup?

A: In that example, the operation is not T, but the composition of T's. In fact the carrier set for the group is (here f is T):

$$\mathbf{S} = \{f^n \mid n \in \mathbb{N}\}$$

This is a binary operation because it has the form ";" : T x T -> T.

Q: Can we discuss this example please? In math we think of R^3 as a vector space (regarded here only with the operation of addition of vectors) with addition of vectors and the zero vector as an identity element. But in physics there is no God given origin of the 3d space R^3. We can choose our origin freely. Is this a semicategory?

A:: What do you want to discuss specifically? A first observation I would make is that your identity element doesn't have to be intended as "origin". Mathematically, it is just an identity element. If you require that your interpretation of the 3D space and the identity element have to correspond to space and origin, you'll have to define an operation which "takes care" of your "new" origin.

Regarding the question “Is this a semicategory”, remember: To specify a semicategory you need to specify:

1. Objects
2. Morphisms
3. Composition of morphisms

And they have to satisfy associativity. What would be these constituents for your question?
Note that in the definition of a semicategory there is no mention of identity element/morphism (which btw are two different things).

If I understand your question correctly, you see objects as vector points in R^3, and morphisms between any two points A,B as the “distance vector” AB. Then composition is sum of distances, right?
Yes right, objects are points in 3d, morphisms are vectors in space with head and tail in any pair of points in space (not only vectors with tail in the origin) and composition is geometric addition of vectors (complete the parallelogram)

GZ: Yes. Where does the origin play a role here?

Ok, updated question: What kind of structure is this? Monoid? Semigroup? Semicategory? Category? Something else?
GZ: I should have answered this in the lecture Q&A. Let me know on Zulip if that’s not the case.

Q: Is a morphism always a function with two arguments? If yes, how does this term make sense: identity morphism ?
A: Let’s be careful with the terminology: a morphism is a morphism, not a function (it is a function e.g. in the specific case of the category of sets and functions, which Jonathan looked at couple of slides ago). The identity morphism on any object X is a morphism X -> X which satisfies the unitality law.

Q: Is Hom(C) always technically a set? In other words, do we require the same consideration we had with respect to the size of the collection of all objects in a category?

A: This is a good question, which is related to calling it “A collection” or “A set”, and to e.g. Russel’s paradox. For now, assume it is a set (this will make sense in our examples).