Applied Compositional Thinking for Engineers

Spring 2021

Why?

Compositionality in engineering

- Because the human brain has limited capacity,
  we **need to rely on abstractions and decomposition**.

- **We need to decompose systems into components.**
  What is a system? What are the components?

> **"A system is composed of components;
> a component is something you understand."**
>
> — Howard Aiken (1900-1973)

- What types of decomposition there are?
- What is abstraction exactly?
- Can we talk about these things generally, without reference to a particular domain?

Applied Compositional Thinking

Applied Category Theory

What Applied Category Theory is for an engineer

- **You, as an engineer, have developed very good mental models** of systems and components in your domain.
  - Some models are formal, some models are “informal”.
  - Some models are computational, some are not.

- **You use the more formal models to design, simulate, analyze your systems**.
  - For example: $\dot{x} = f(x)$

- The formal models also **help you communicate and share your ideas**.

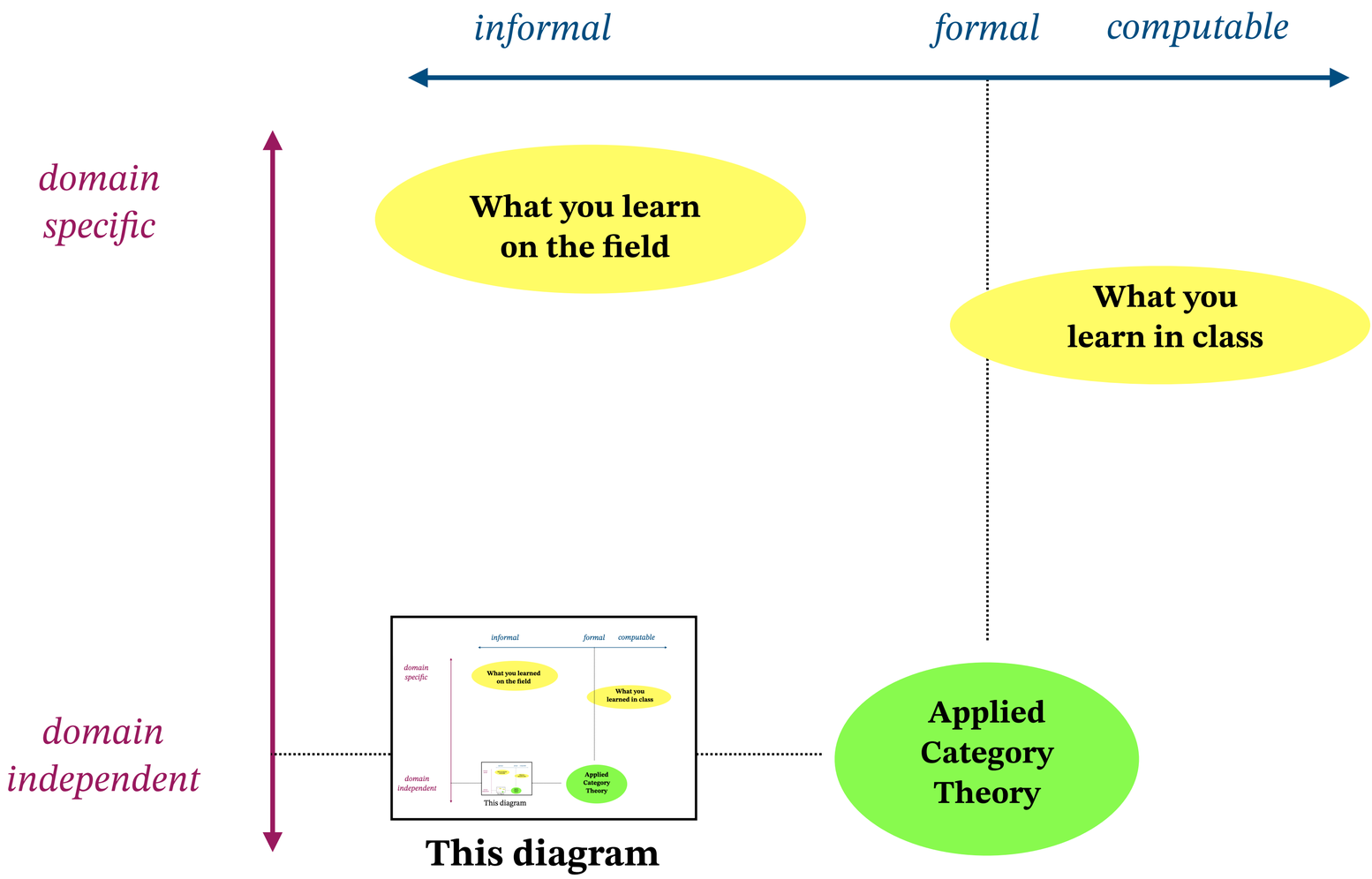
- Think of the **mathematics** you already know as **a set of patterns** to describe the world.

- Now, unfortunately you are an engineer **in the 21st century:**
  - The engineers before us solved all the easy problems!
  - The new problems to solve **have to deal with very complex systems.**

- **Applied Category Theory is an updated set of mathematical patterns that is particularly suited to describe large systems** with complex interaction/interconnection/decomposition features.

The case for Applied Category Theory

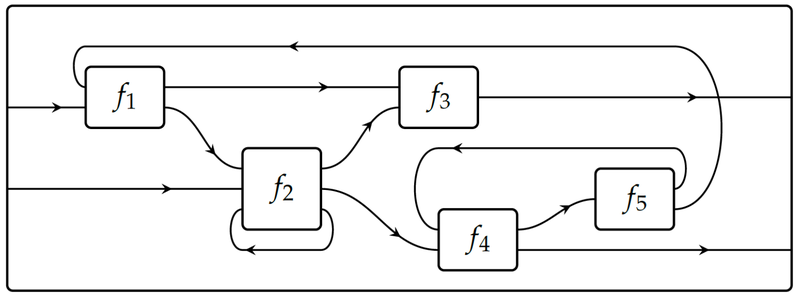
- Applied Category Theory is **formal** but **domain-independent**.



The case for Applied Category Theory

- **What can Applied Category Theory do for you:**
  - It is a **mathematical theory** equipped with theorems, methods, and **"libraries" of re-usable concepts/connections.**
  - It s a **formal and visual language** for reasoning/thinking and design.

- **Example**: These **"boxes and wires" diagrams** (representing something called "*traced monoidal categories*") have wildly different semantics according to the domain.

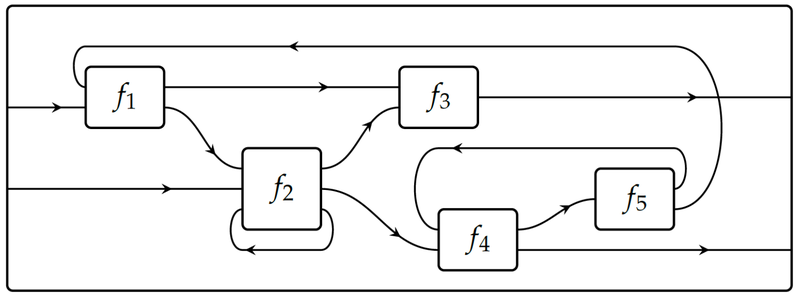


  - However: **in many cases, the rules about composition are always the same**.
  - We can study the concept and its implications abstractly, then re-particularize the conclusions to the domain of interest.

The case for Applied Category Theory

- **What Applied Category Theory cannot do:**
  - It cannot, by itself, solve domain specific problems.
  - However, it can help you formulate the problems well.

- **The catch:**
  - You will need to **invest *more* work at the beginning** in the definitions, formalization and modelling choices for your problem, but...
  - **Payoff comes later** in that things "work out" and that useful properties can be proven easily/quickly (they come for "cheap" or "free").



Obstacles to learning ACT as engineers

- While it is not particularly difficult, the **math can be unfamiliar** to engineers.
  - Engineering background is heavy on continuous math (analysis, ...), very light on discrete mathematics (algebra, ...) that are prerequisites for applied category theory.

- The **materials** you can currently find are **not particularly tailored for engineers**.
  - Typical audiences:

    Mathematicians

    Computer scientists

    Software engineers

- **No in-depth discussion of actual examples** relevant to engineers.

- The typical **curriculum** is at the same time **too broad** and **not broad enough**.
  - There are basic topics in ACT that engineers do not need.
  - There are topics that mathematicians consider very niche that have strong appeal for engineers.

- **We started the class to fix the situation.**