

Applied Compositional Thinking for Engineers



Spring 2021

Composition

How do I compose thee?

Let me count the ways.



Oct. 24, 1961

G. K. CHRISTIANSEN

,005,282

TOY BUILDING BRICK

Filed July 28, 1958

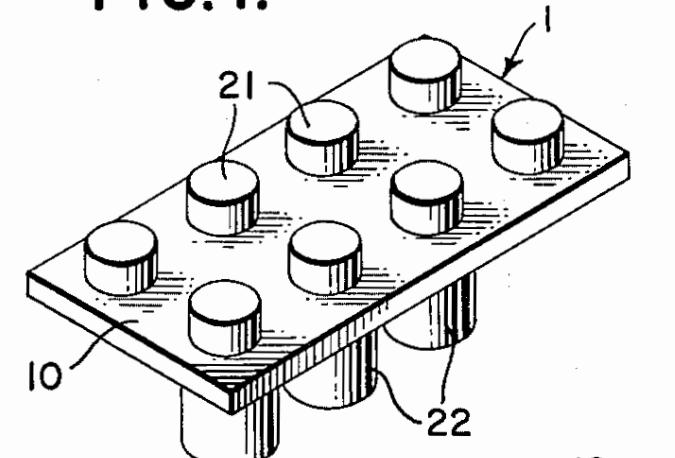


FIG. 2.

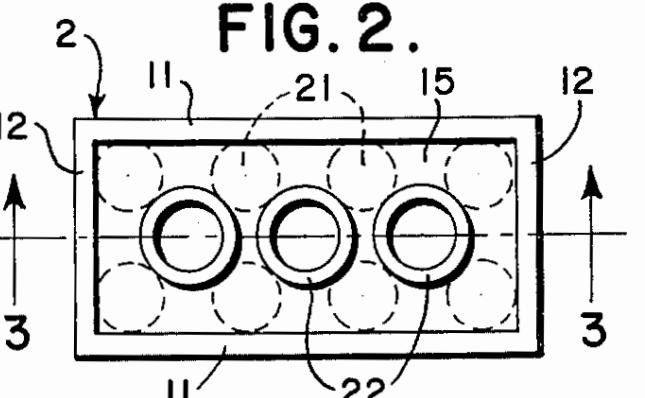


FIG. 3.

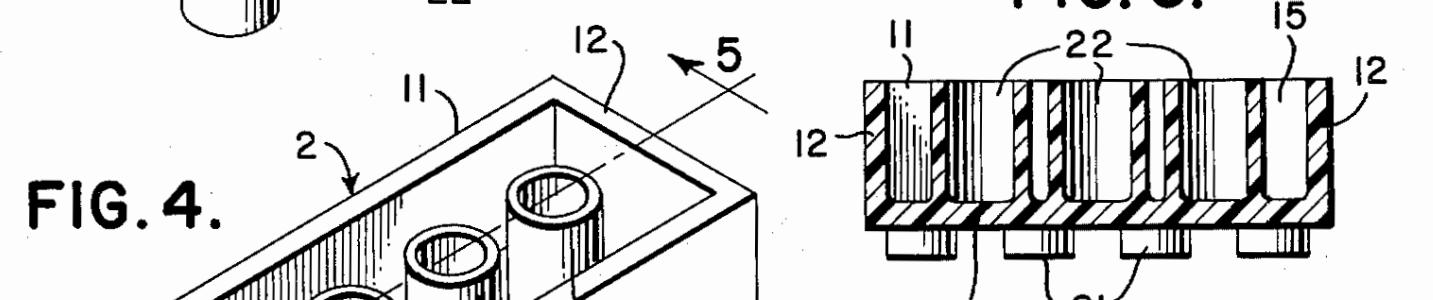


FIG. 4.

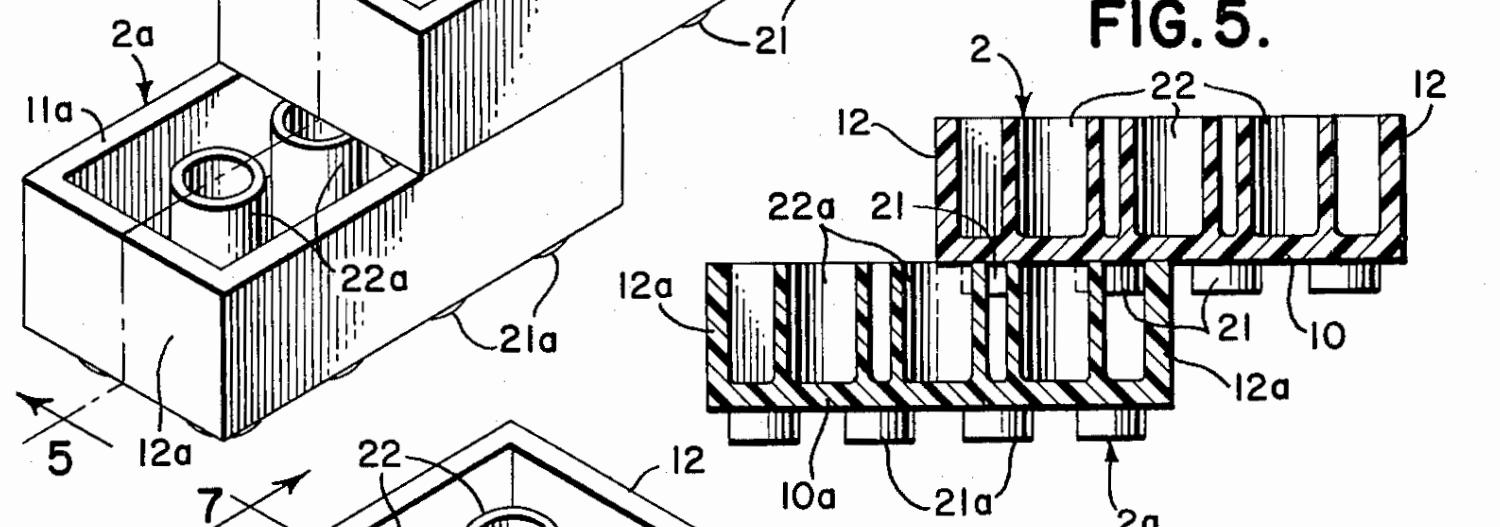


FIG. 5.

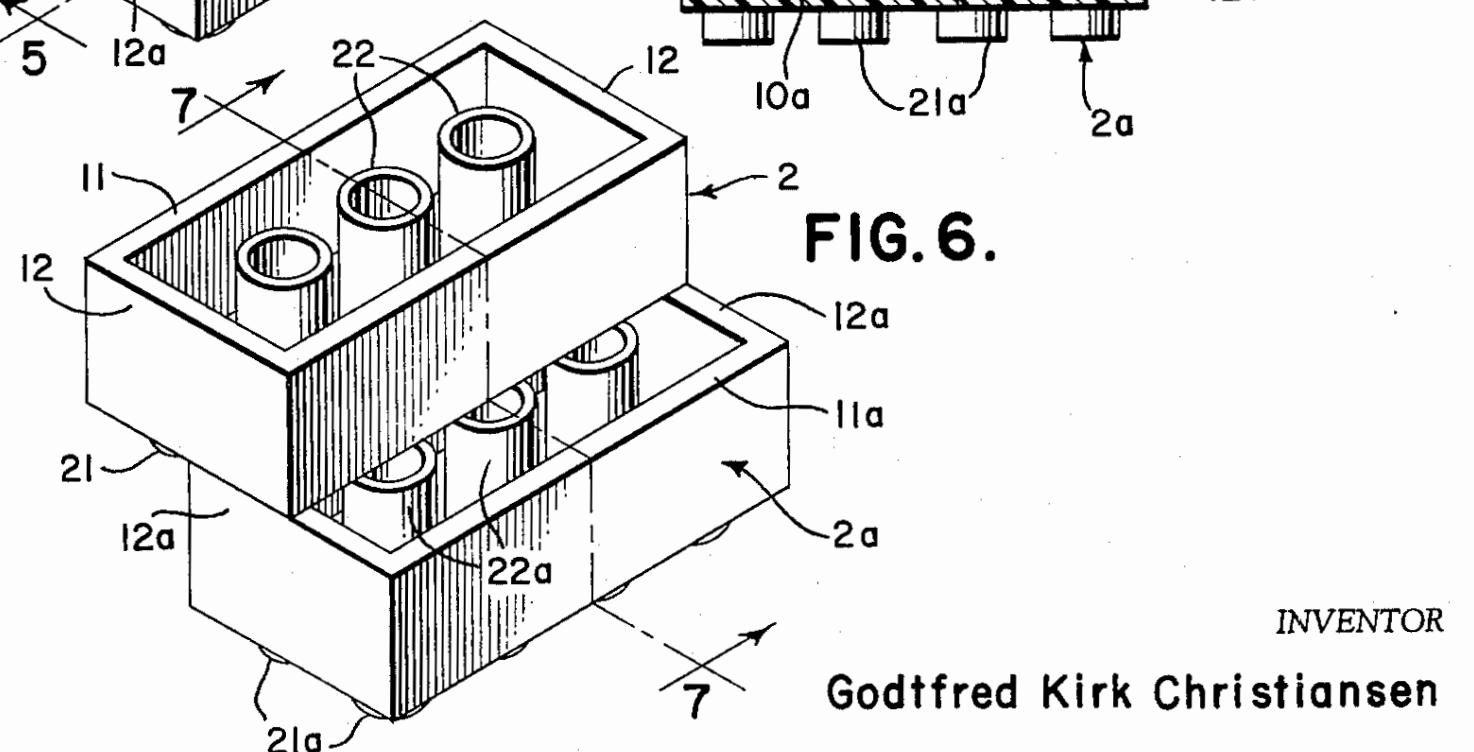


FIG. 6.

INVENTOR

Godtfred Kirk Christiansen

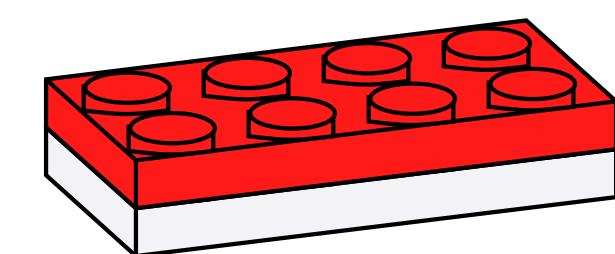
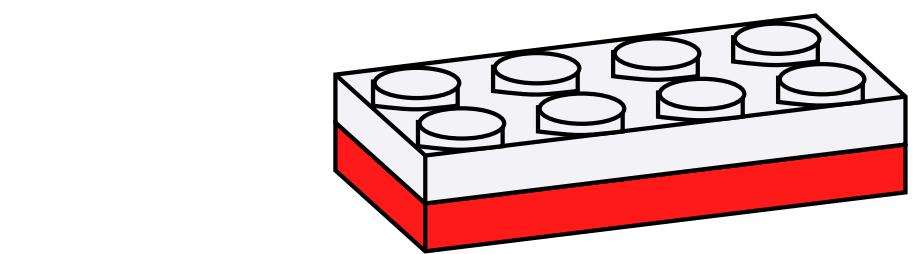
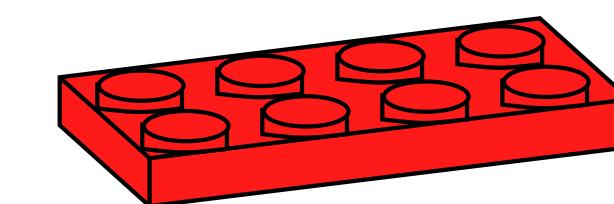
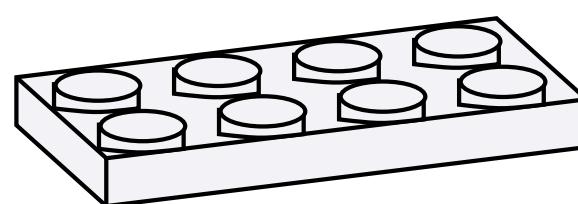
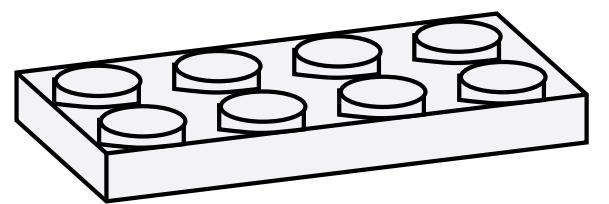
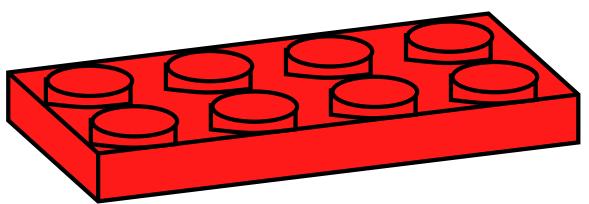
^Y Stevens, David, Muller & Mosher
ATTORNEYS



Composition recipes

ingredient ingredient ingredient

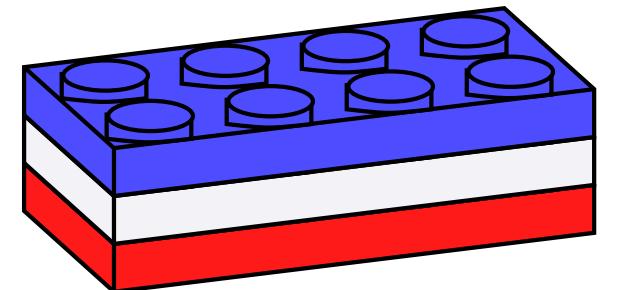
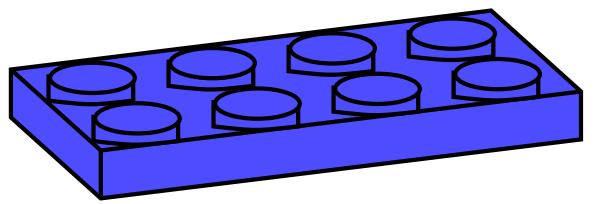
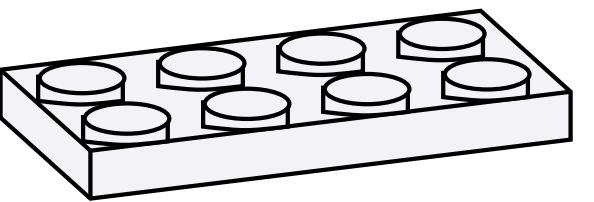
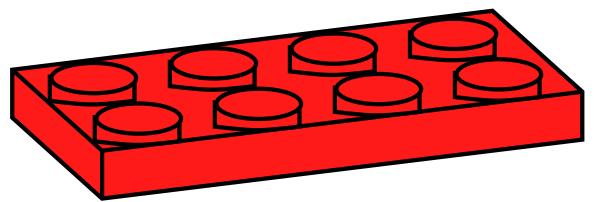
result result



Composition recipes

ingredient ingredient ingredient

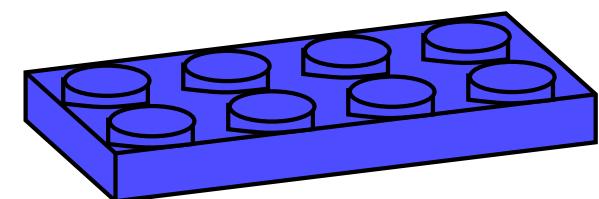
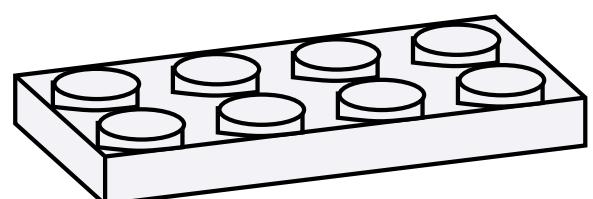
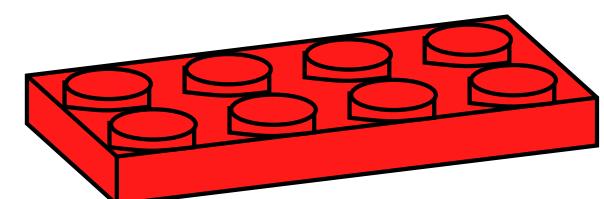
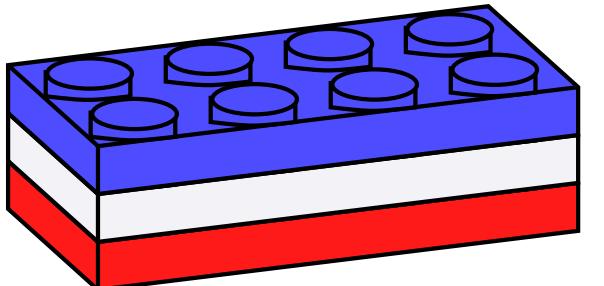
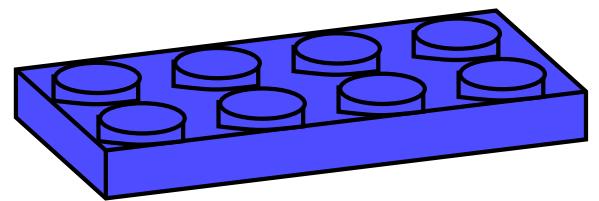
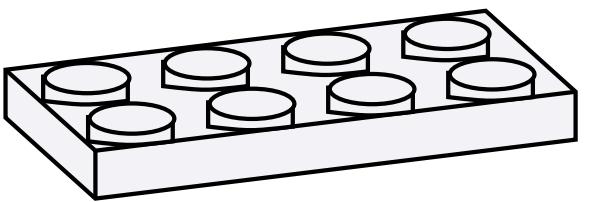
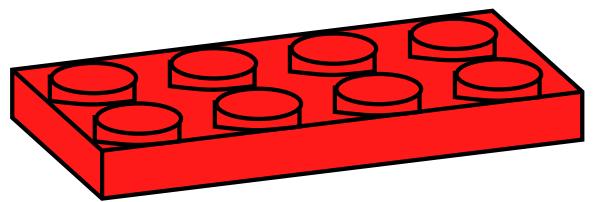
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Composition recipes

ingredient ingredient ingredient

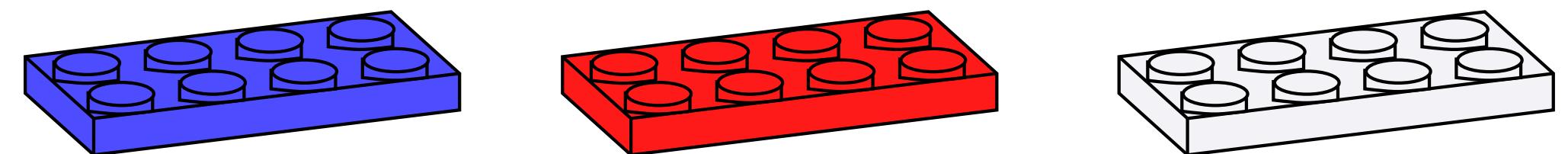
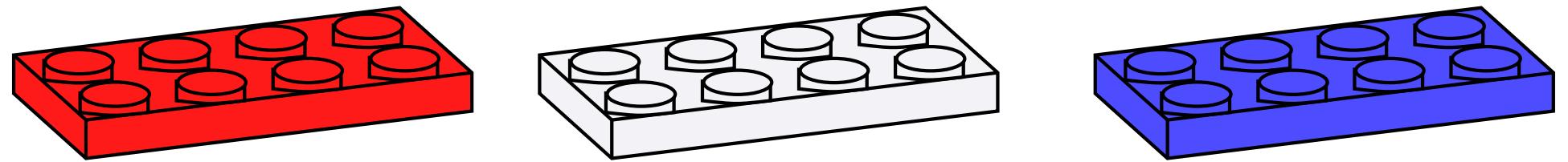
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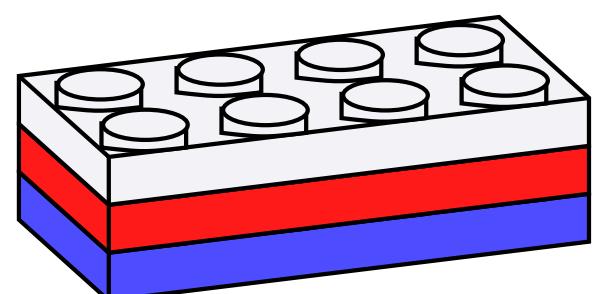
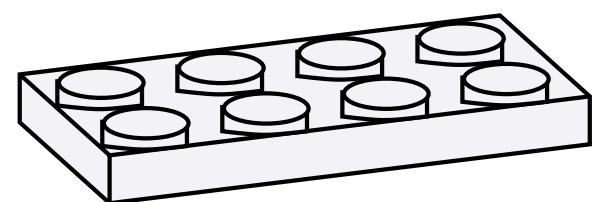
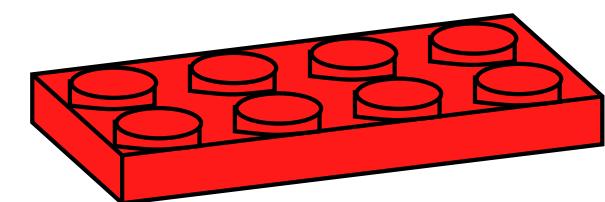
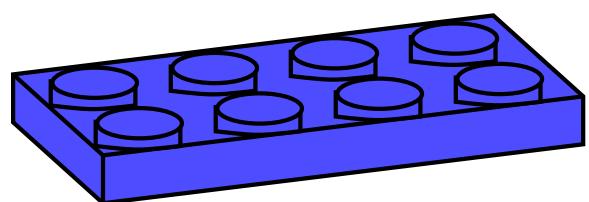
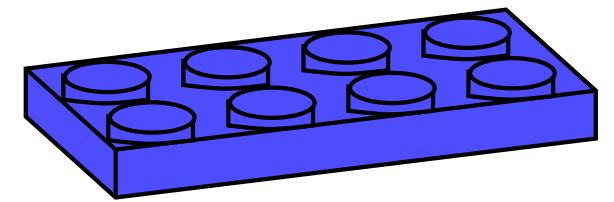
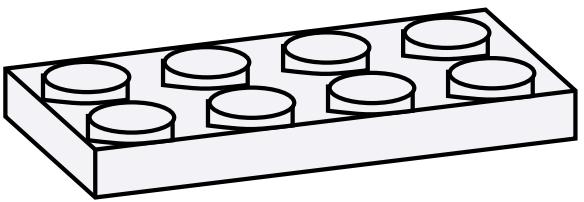
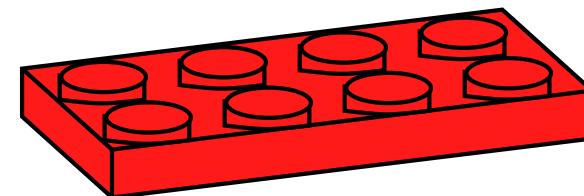
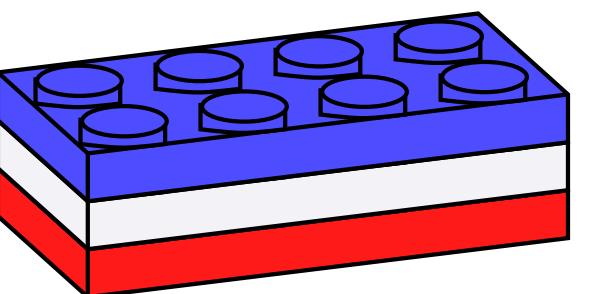
result result



Composition recipes

ingredient ingredient ingredient

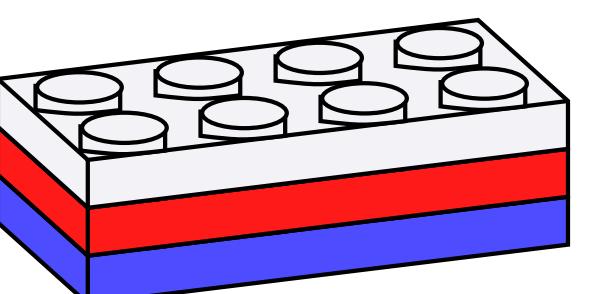
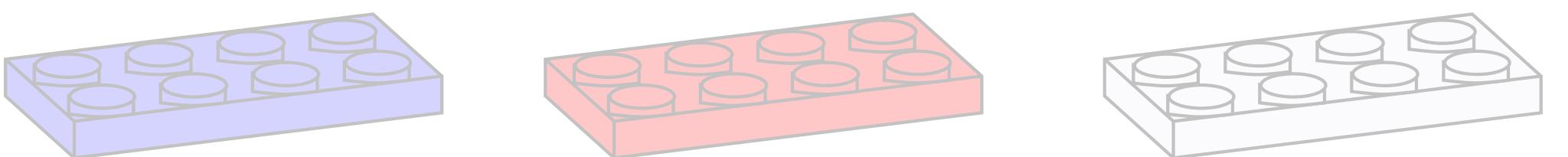
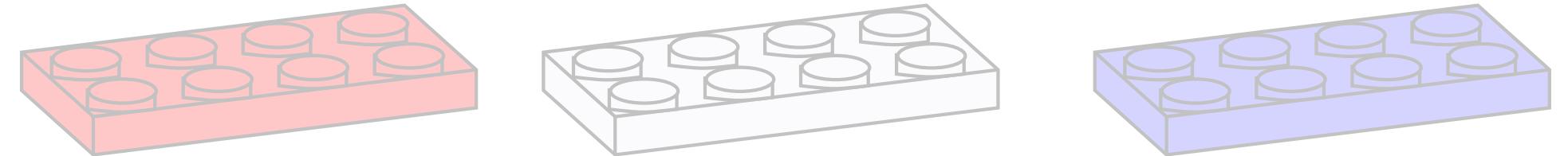
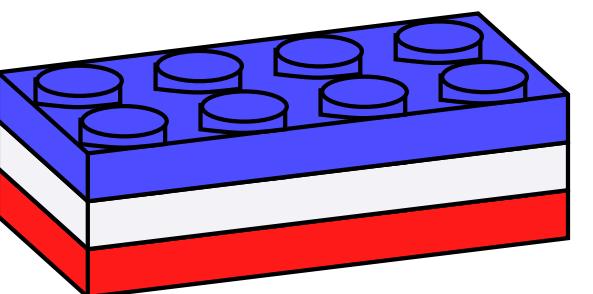
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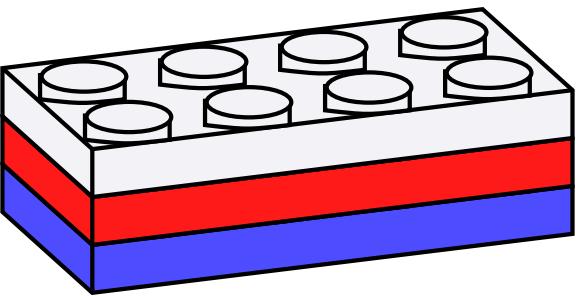
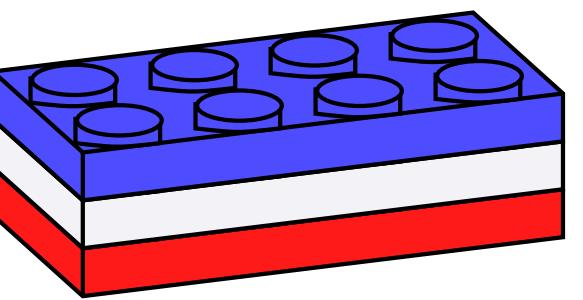
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Composition recipes

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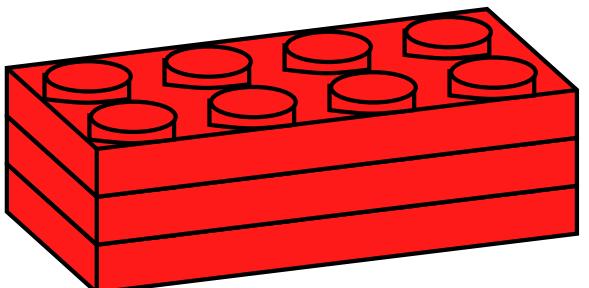
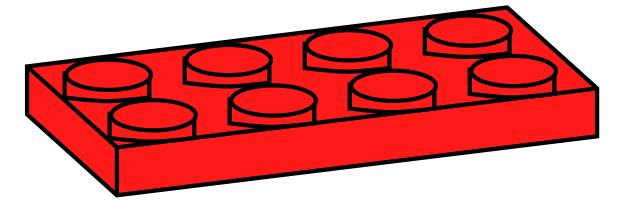
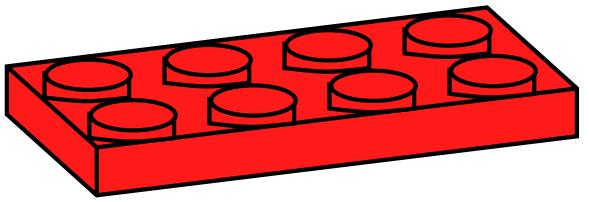
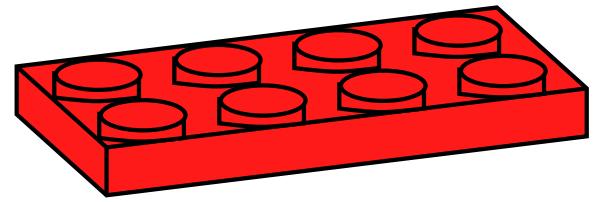
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Composition recipes

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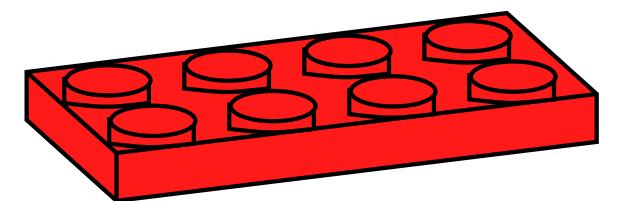
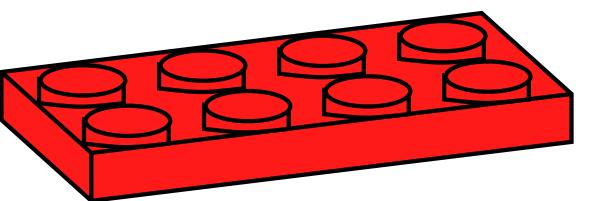
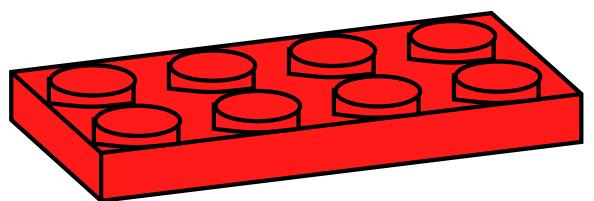
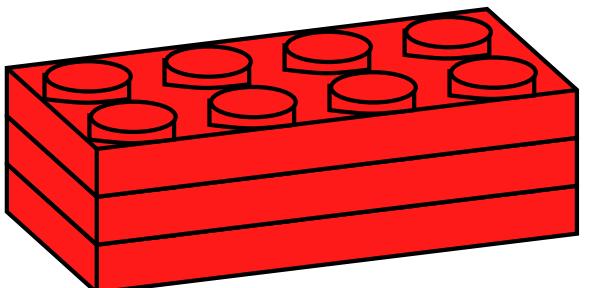
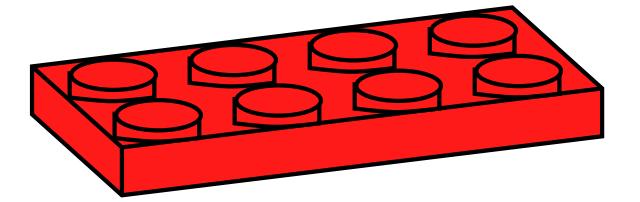
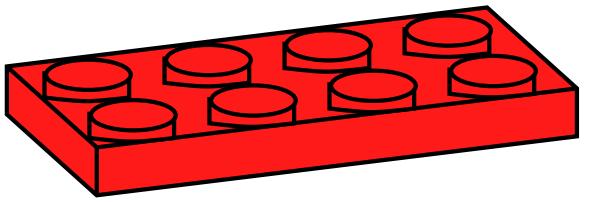
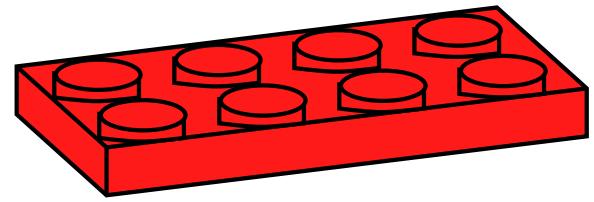
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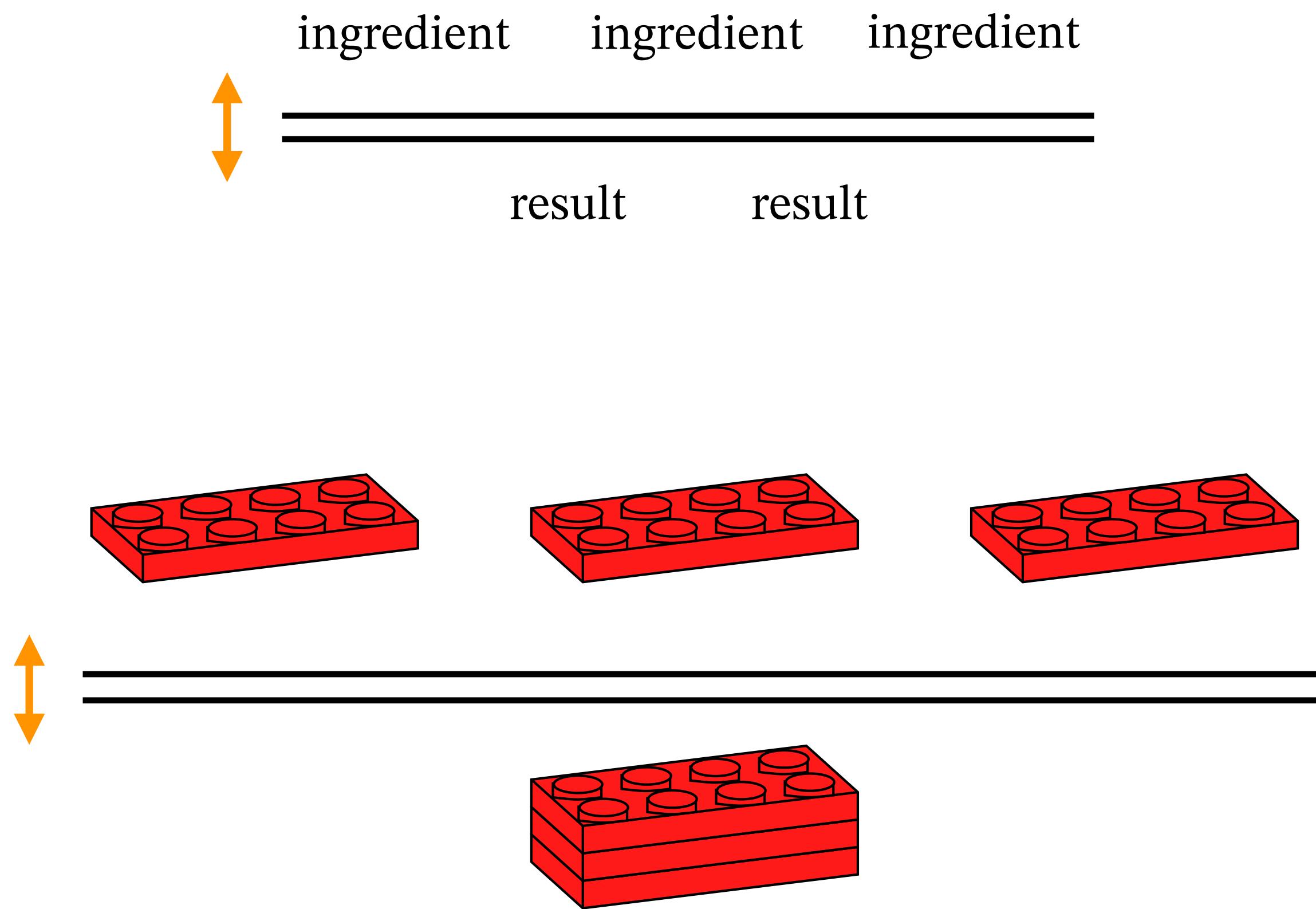
Composition recipes

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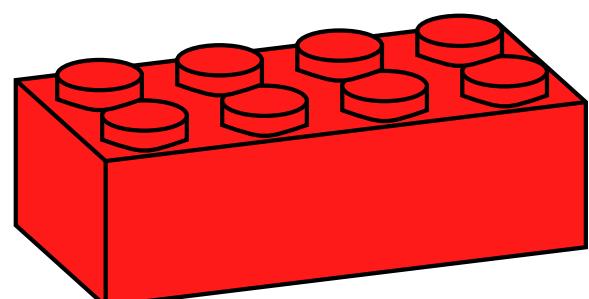
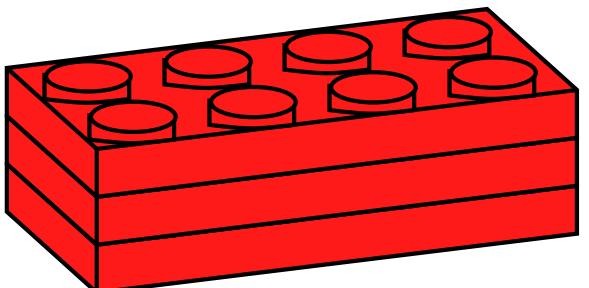
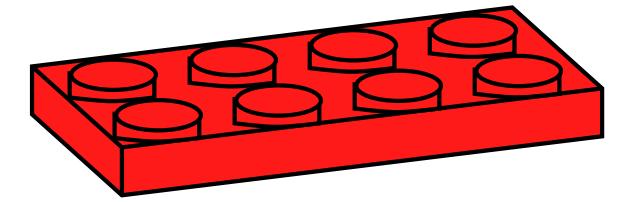
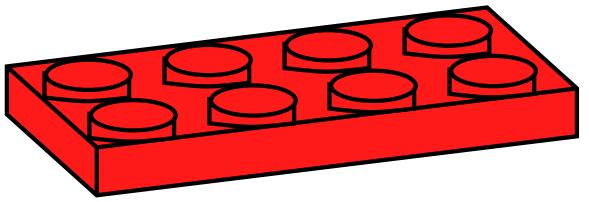
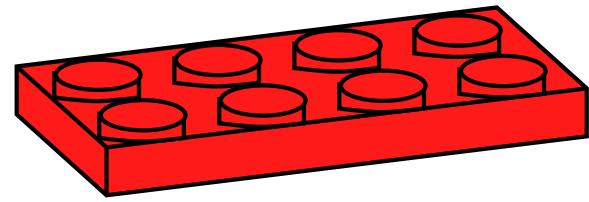
Composition recipes



Composition recipes

ingredient ingredient ingredient

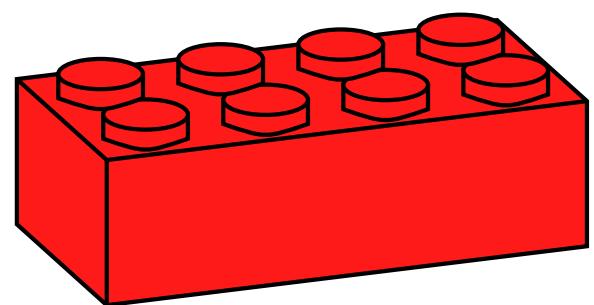
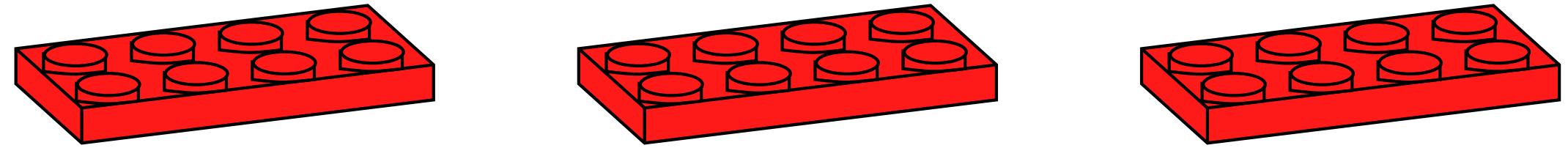
result result



Composition recipes

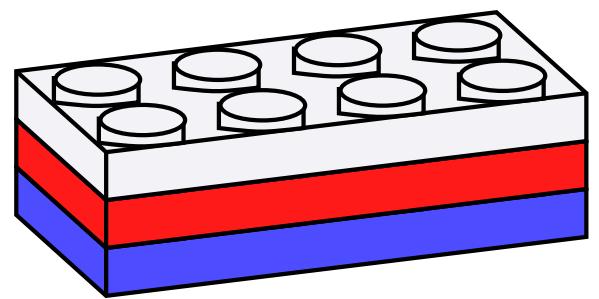
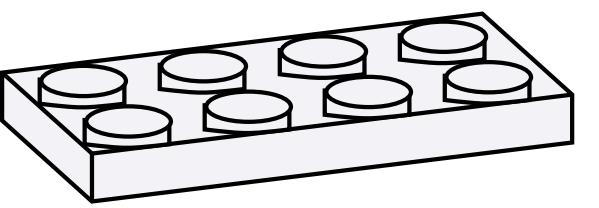
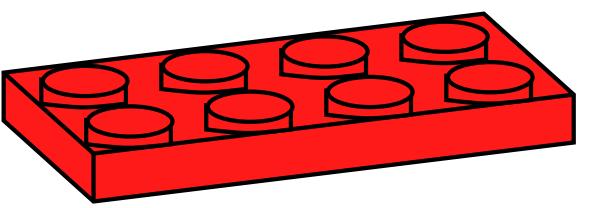
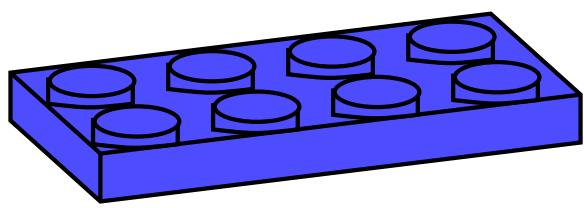
ingredient ingredient ingredient

result result



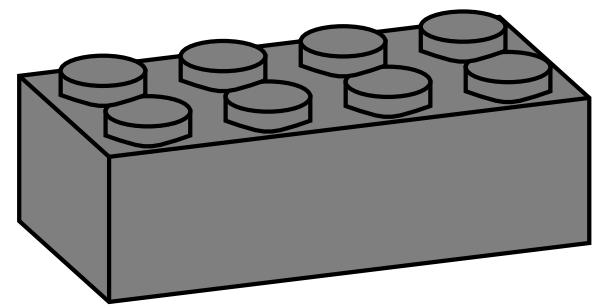
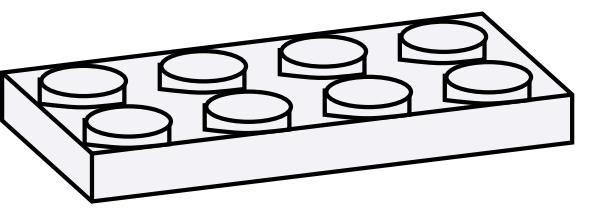
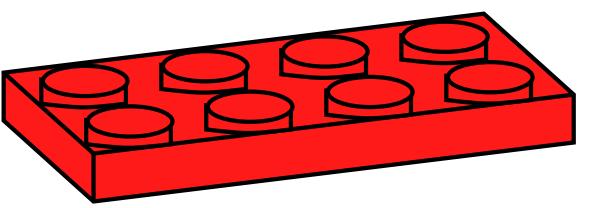
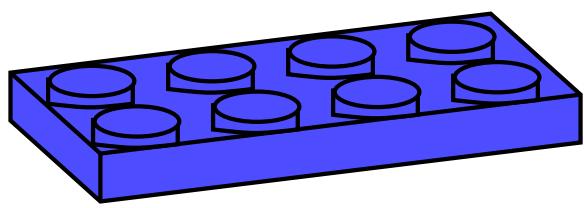
Lego purists

- ▶ Lego purists don't like mixing colors!

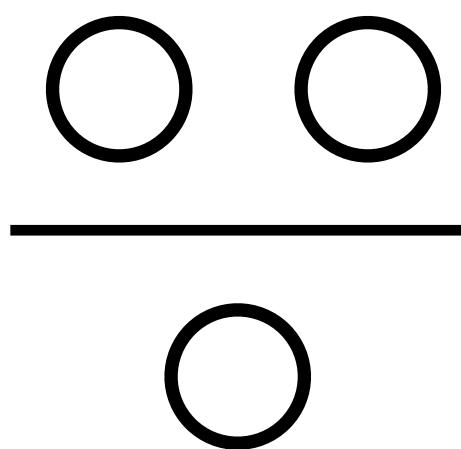
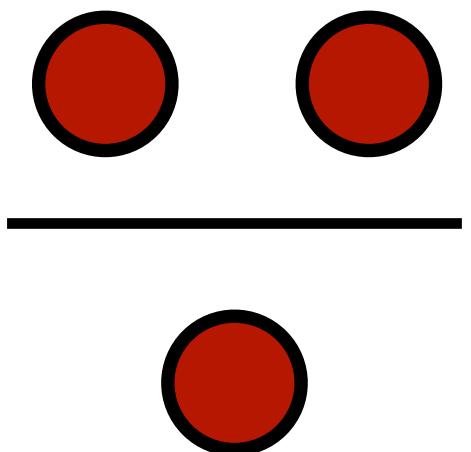


Lego purists

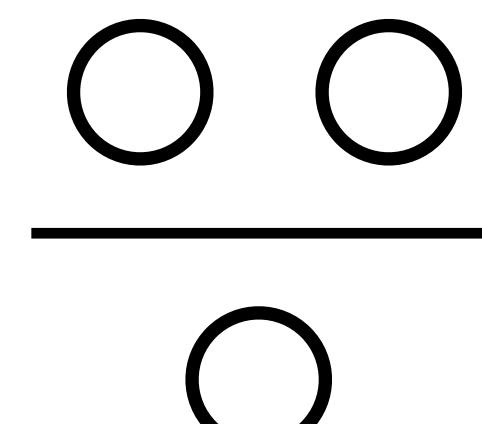
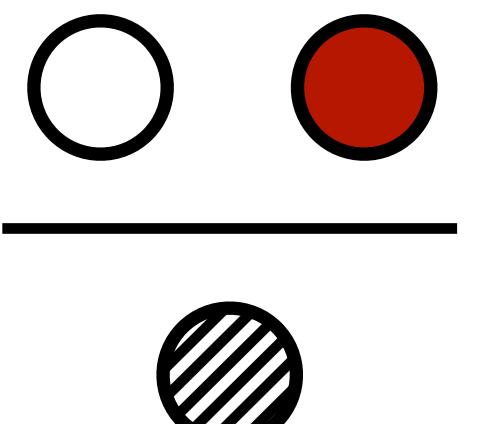
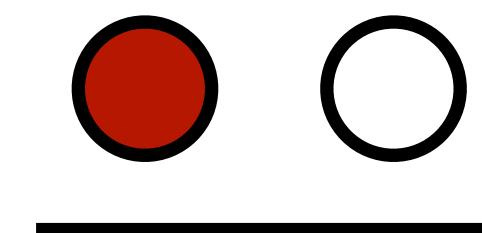
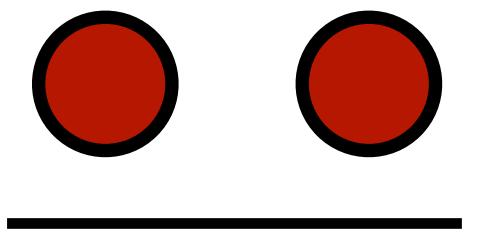
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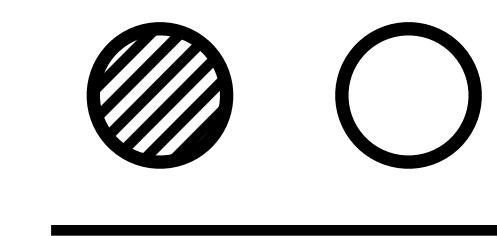
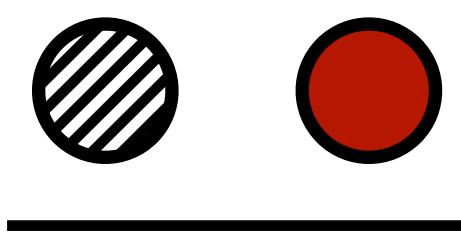
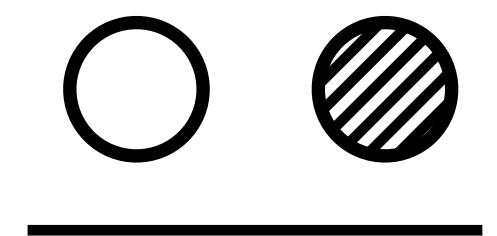
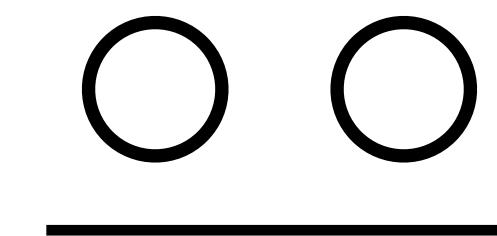
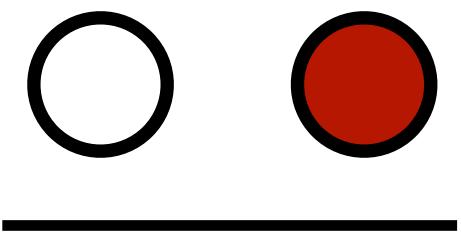
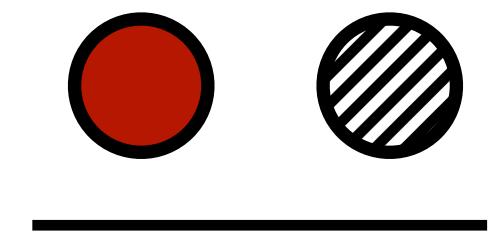
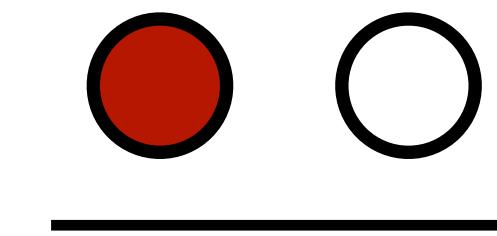
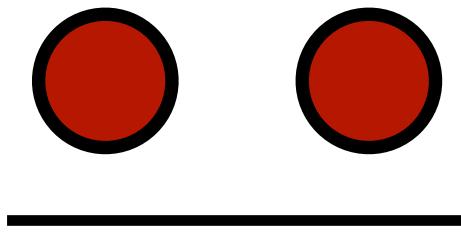
Color mixing for Lego purists



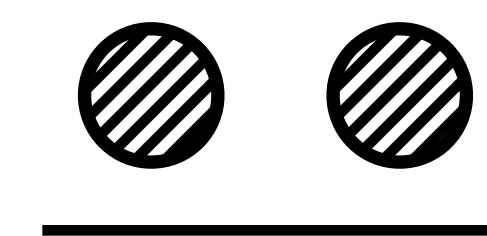
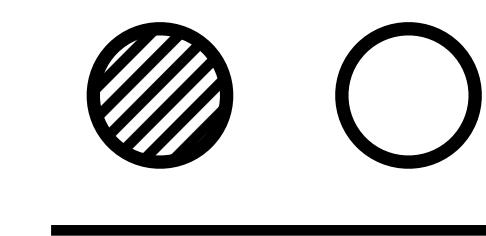
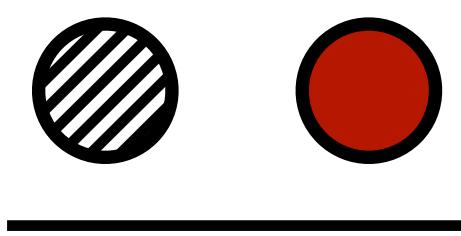
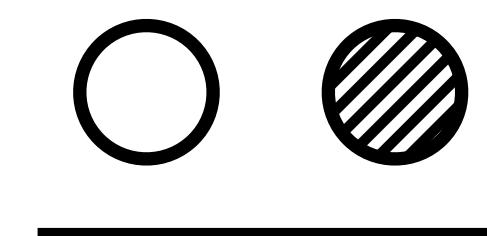
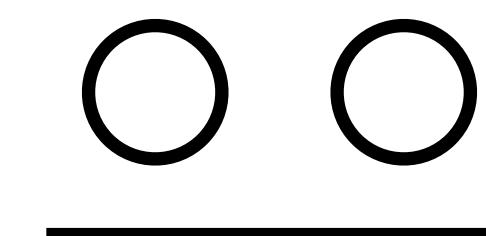
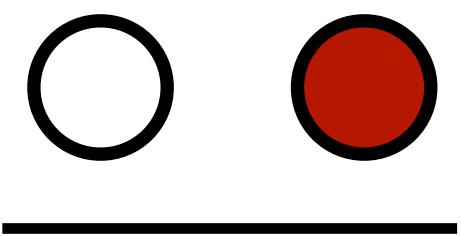
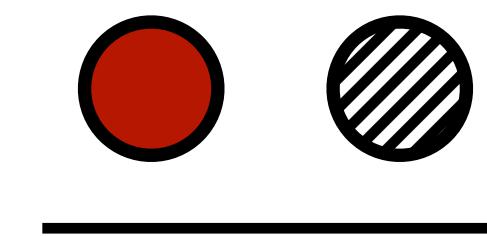
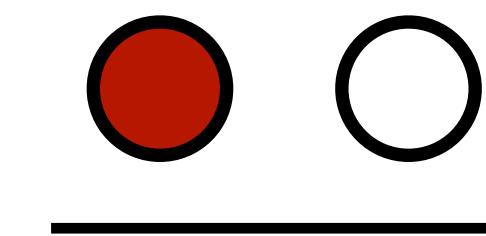
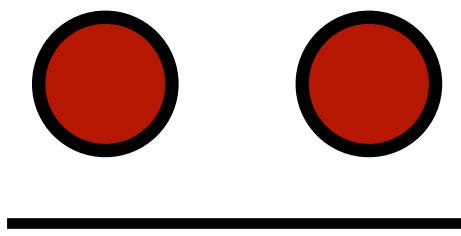
Color mixing for Lego purists



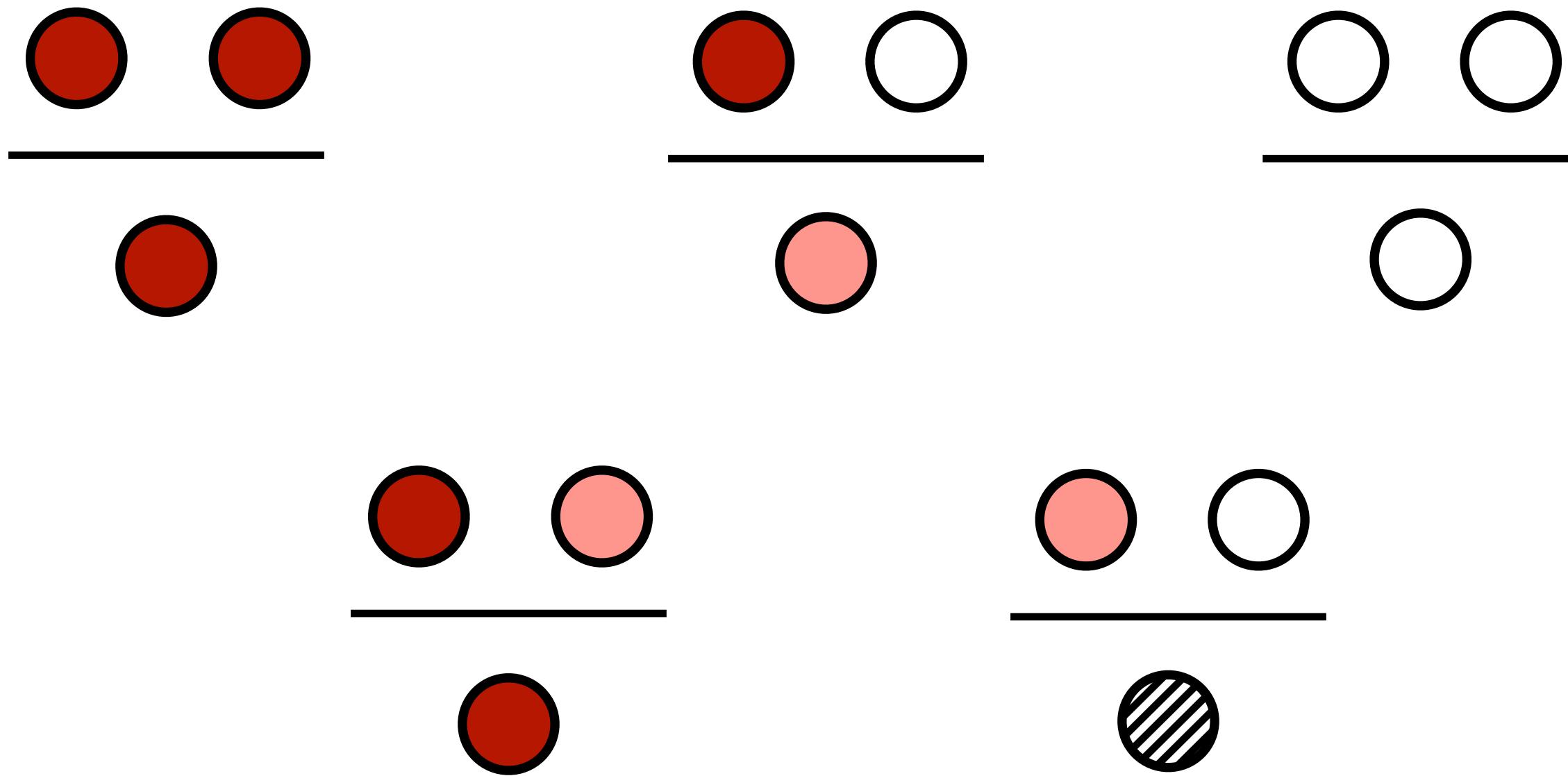
Color mixing for Lego purists



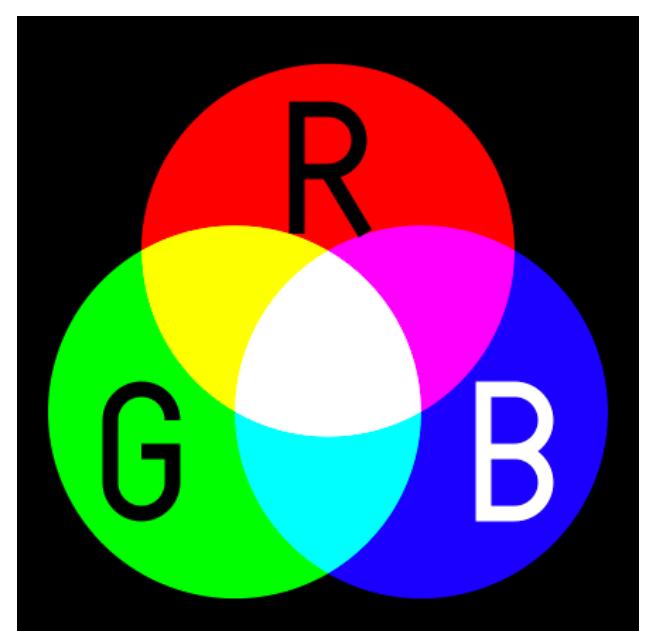
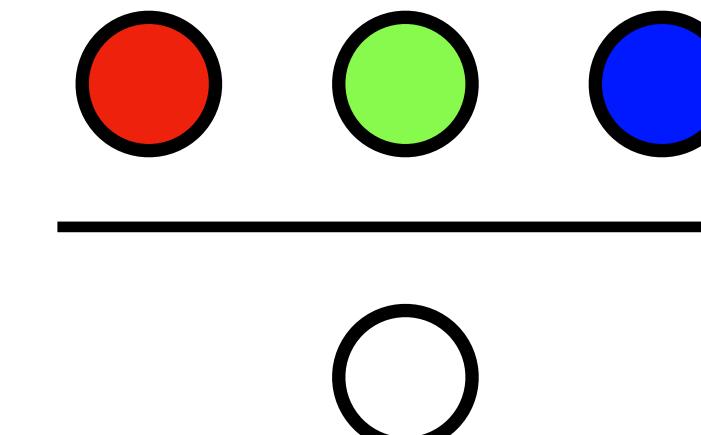
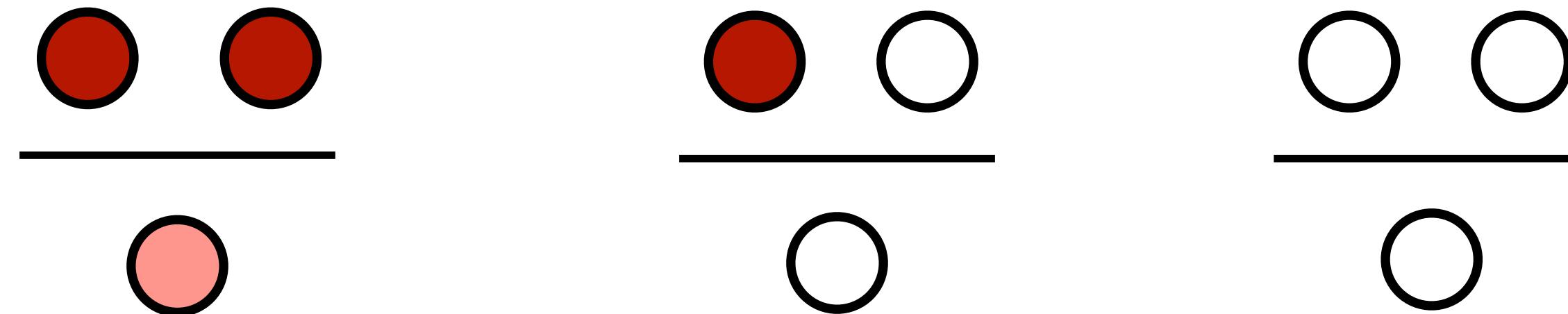
Color mixing for Lego purists



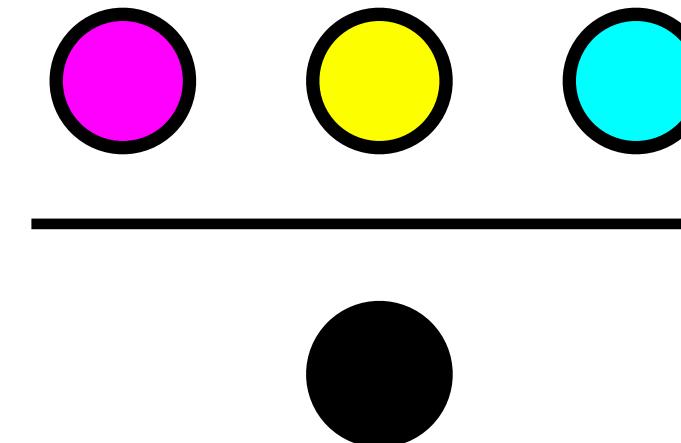
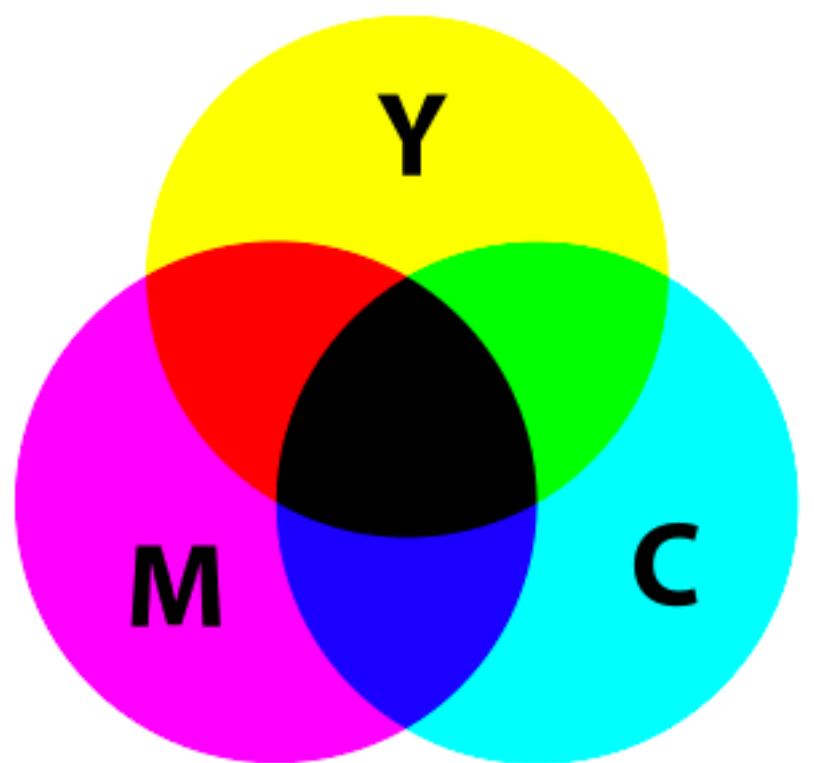
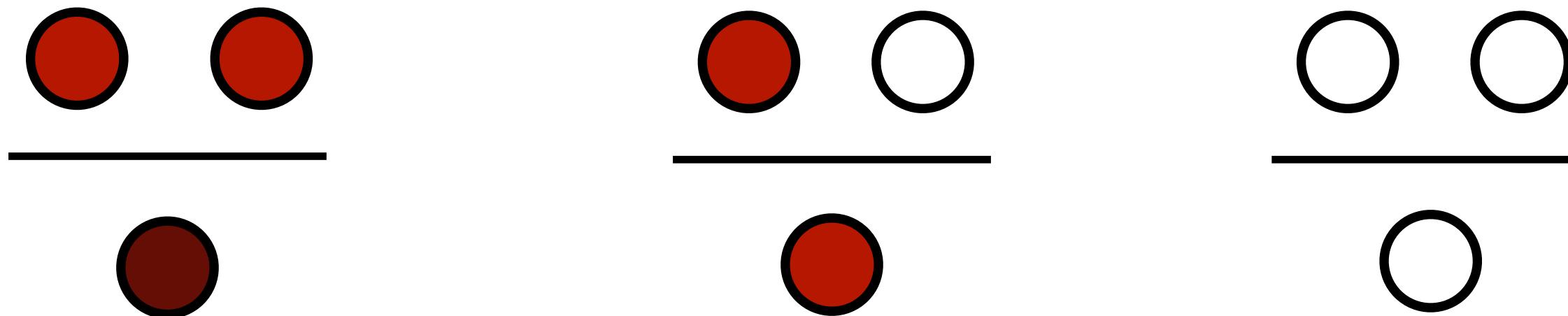
Mixing paint



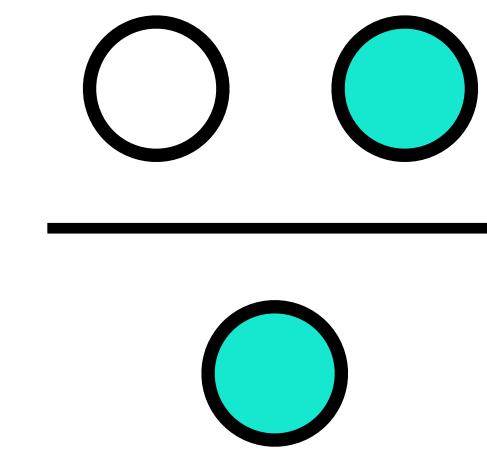
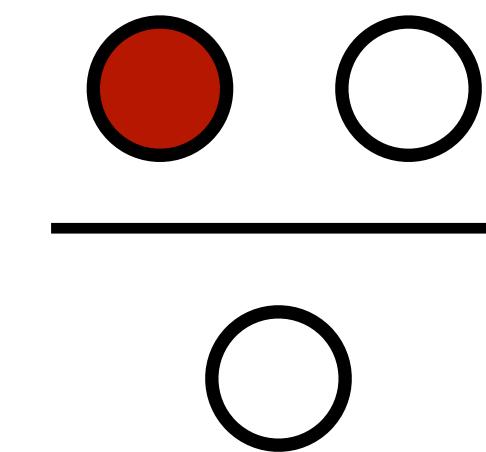
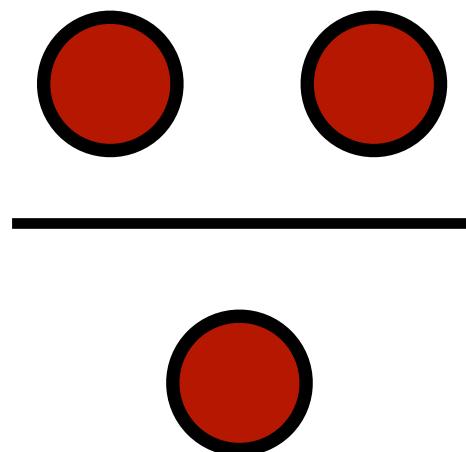
Additive colors (RGB)



Subtractive colors (CMY)



Just paint over it



Quoting

$$\begin{array}{r} \textcolor{red}{\bullet} \\ \hline \textcolor{blue}{\bullet} \end{array} \qquad \begin{array}{r} \textcolor{blue}{\bullet} \\ \hline \textcolor{green}{\bullet} \end{array}$$

$$\begin{array}{r} \textcolor{red}{\bullet} \\ \hline \textcolor{green}{\bullet} \end{array}$$



Abstraction

$$\begin{array}{r} \textcolor{red}{\bullet} \\ \hline \textcolor{blue}{\square} \end{array} \quad \begin{array}{r} \textcolor{blue}{\square} \\ \hline \textcolor{green}{\bullet} \end{array}$$

$$\begin{array}{r} \textcolor{red}{\bullet} \\ \hline \textcolor{green}{\bullet} \end{array}$$



Theorems

$$\frac{\square}{\square}$$

$$\frac{\square}{\square}$$

$$\frac{\square}{\square}$$

$$\frac{\square}{\square}$$



Commutativity

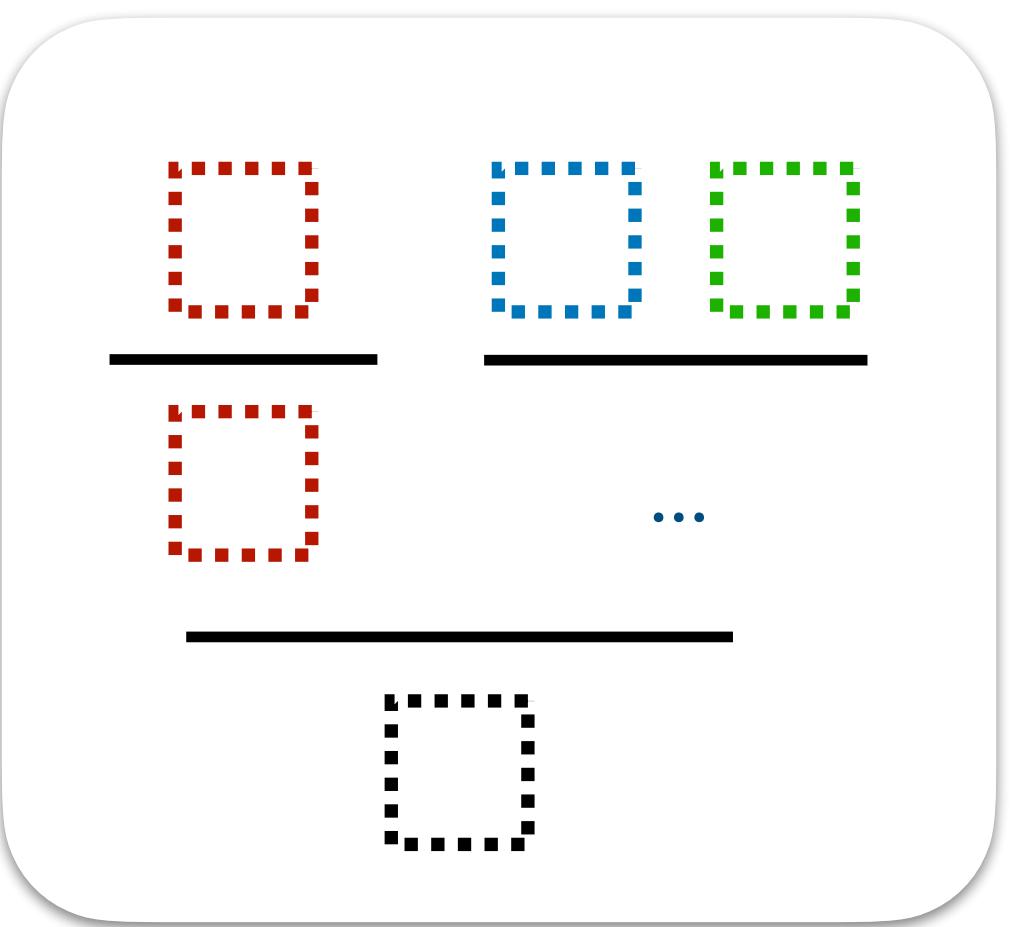
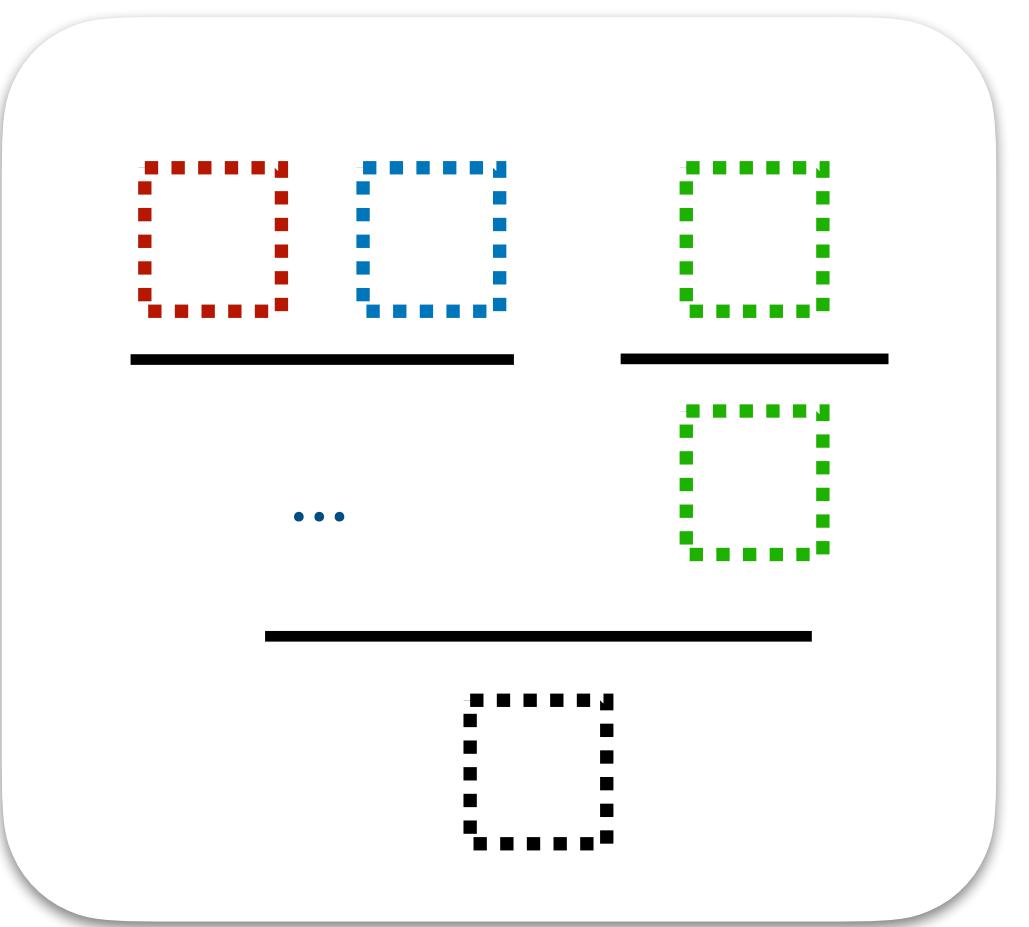
$$\begin{array}{r} \textcolor{red}{\square} \textcolor{blue}{\square} \\ \hline \textcolor{black}{\square} \end{array}$$

=====

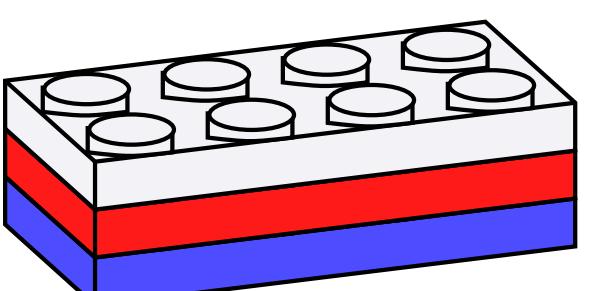
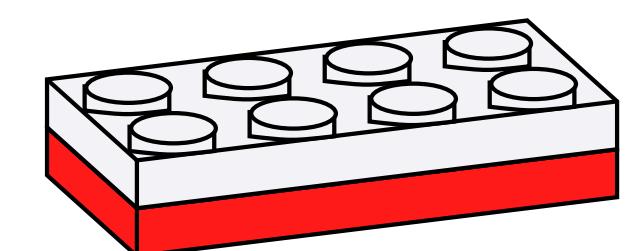
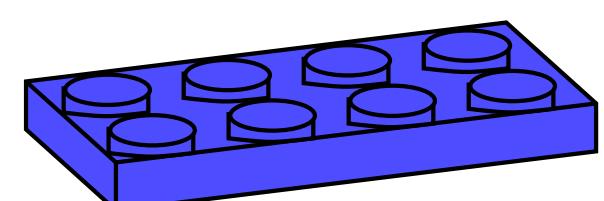
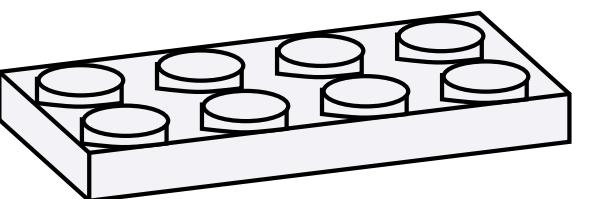
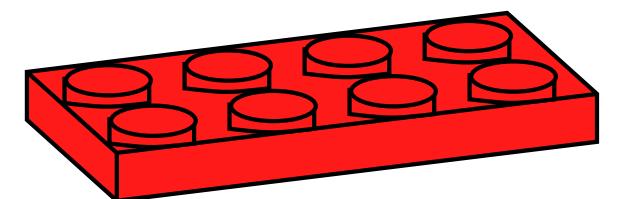
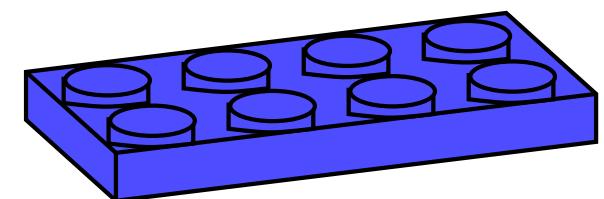
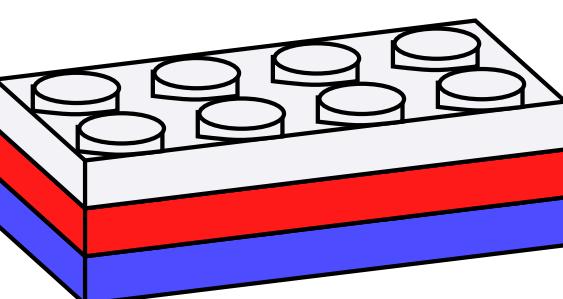
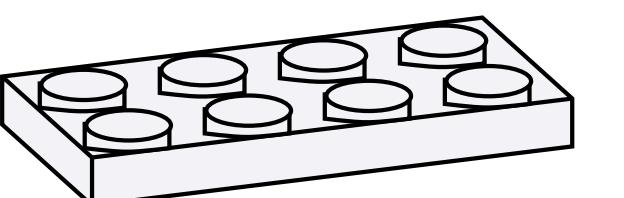
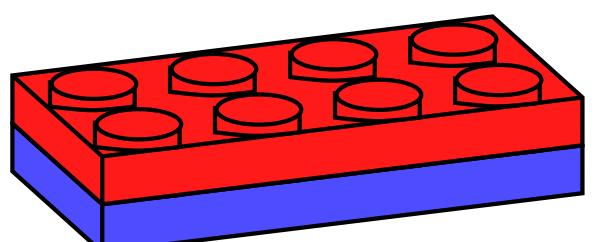
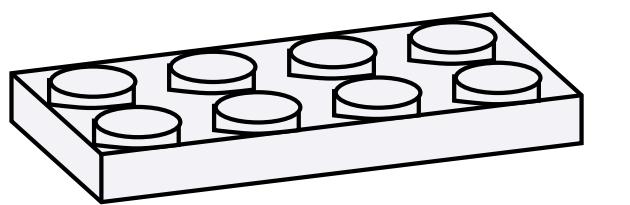
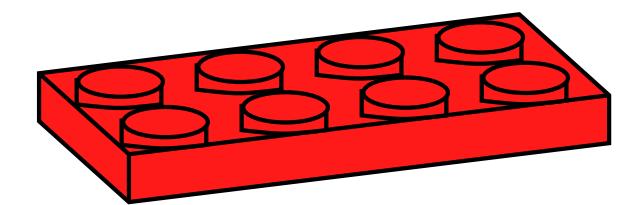
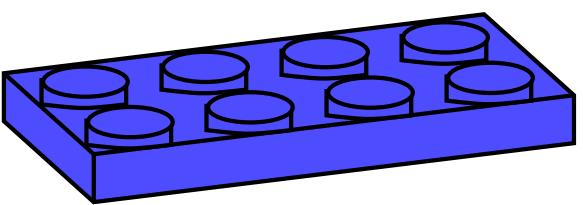
$$\begin{array}{r} \textcolor{blue}{\square} \textcolor{red}{\square} \\ \hline \textcolor{black}{\square} \end{array}$$



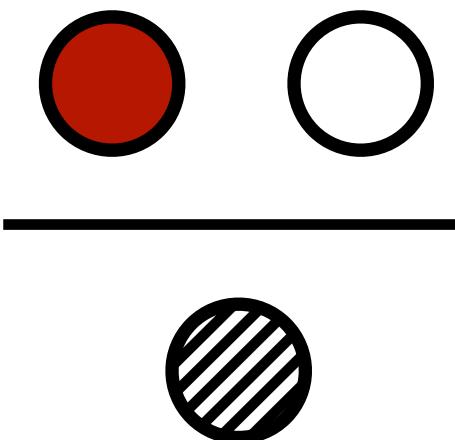
Associativity



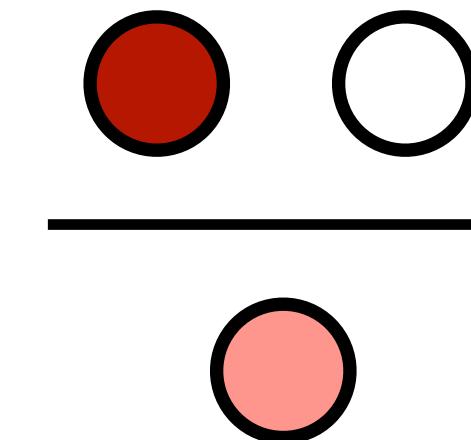
Associativity



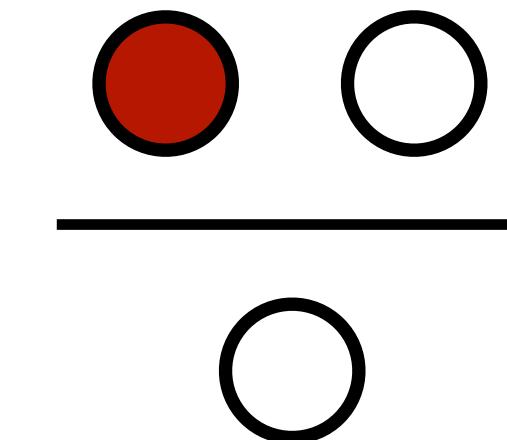
Lego purist



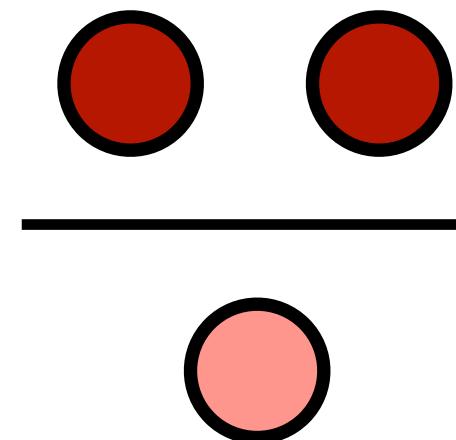
Mixing paint



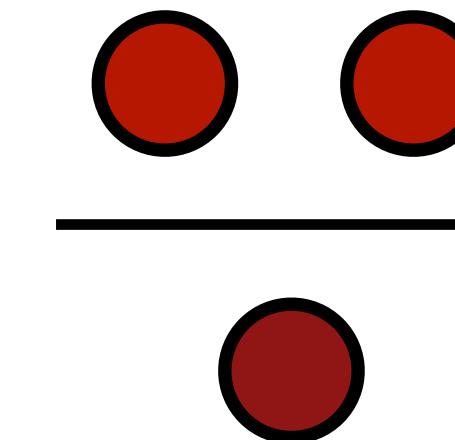
Apply paint



Additive



Subtractive



commutative?

yes

yes

no

yes

yes

associative?

yes

no

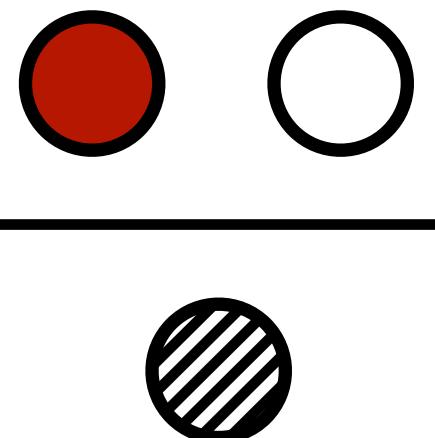
yes

yes

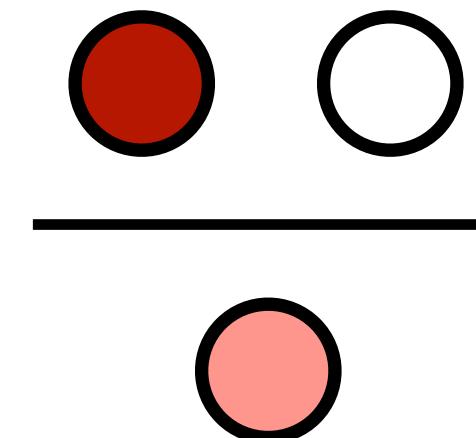
yes



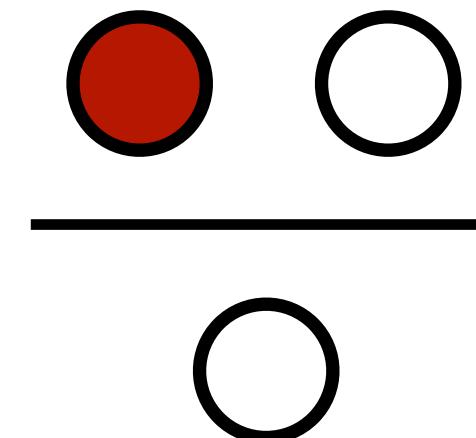
Lego purist



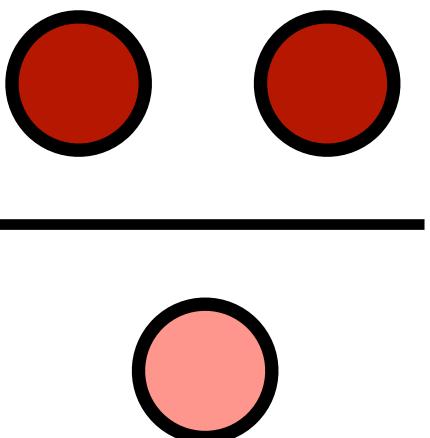
Mixing paint



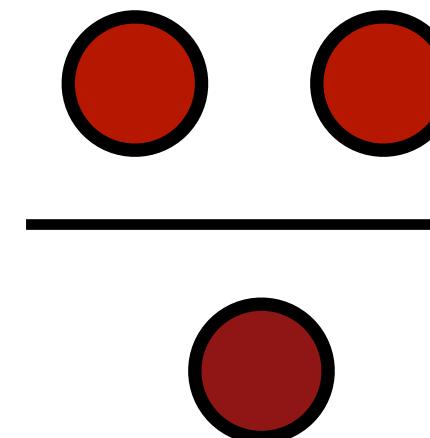
Apply paint



Additive



Subtractive



commutative?

yes

yes

no

yes

yes

associative?

yes

no

yes

yes

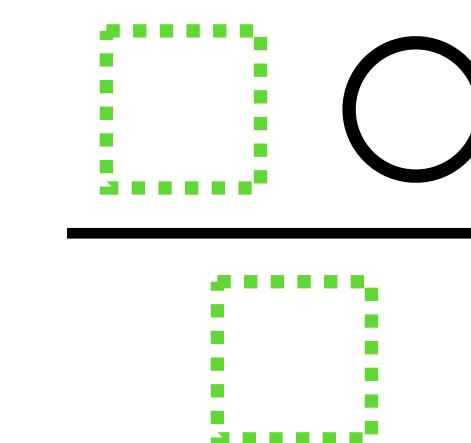
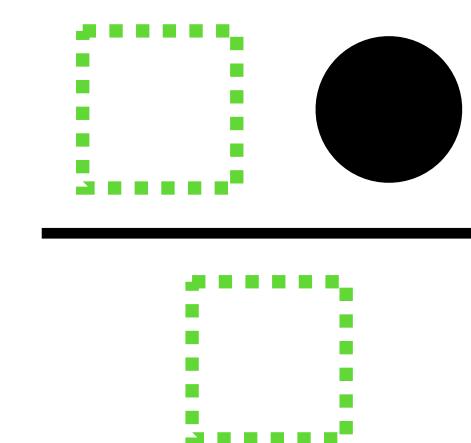
yes

neutral element?

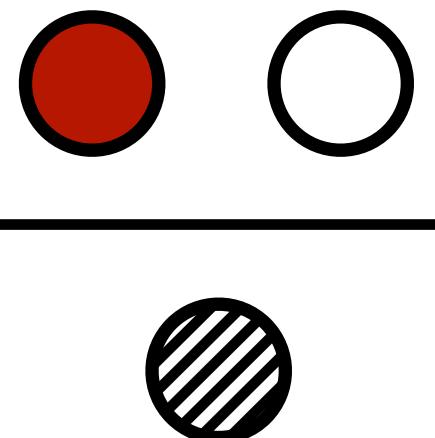
no

no

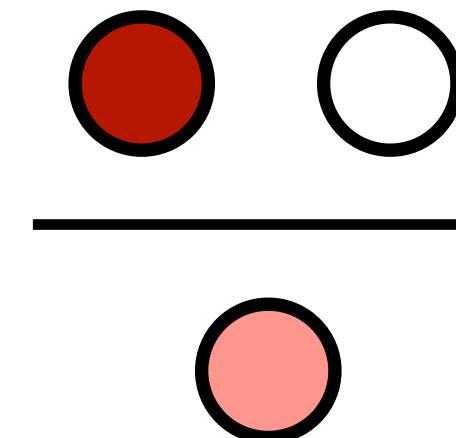
no



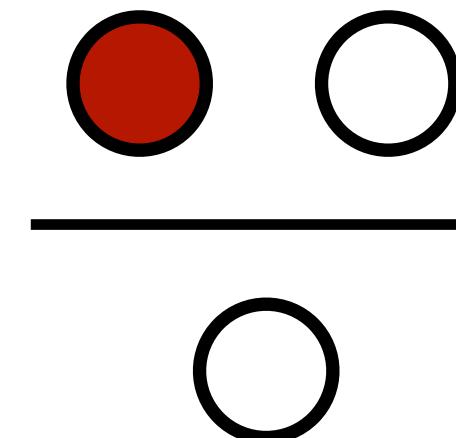
Lego purist



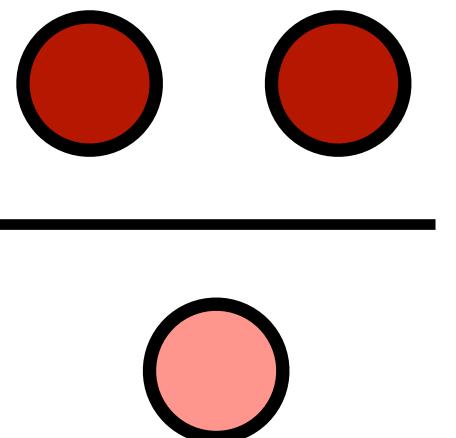
Mixing paint



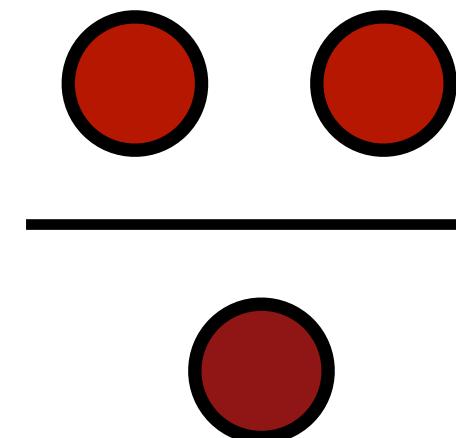
Apply paint



Additive



Subtractive



commutative?

yes

yes

no

yes

yes

associative?

yes

no

yes

yes

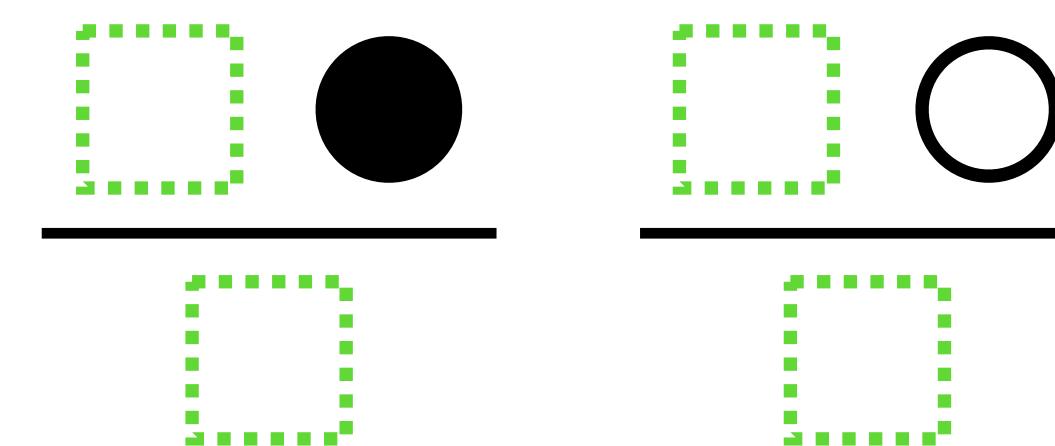
yes

neutral element?

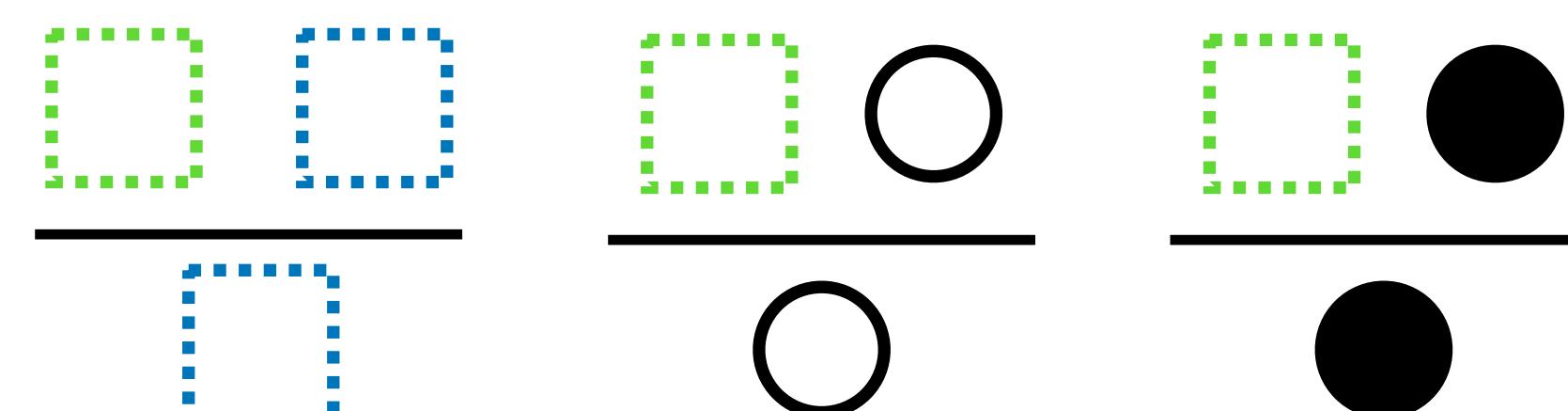
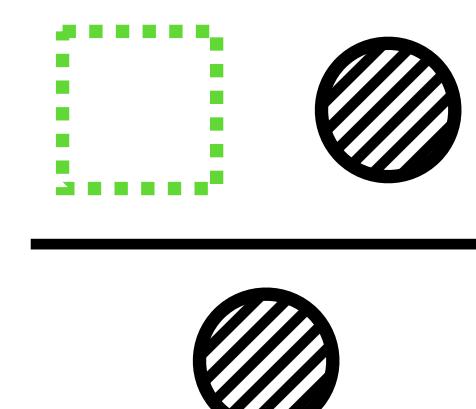
no

no

no

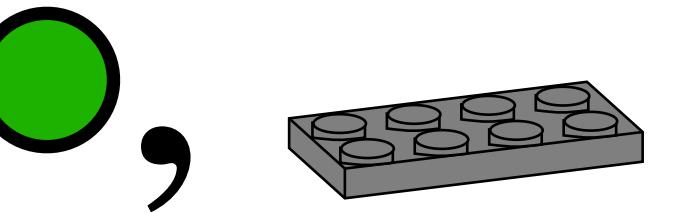
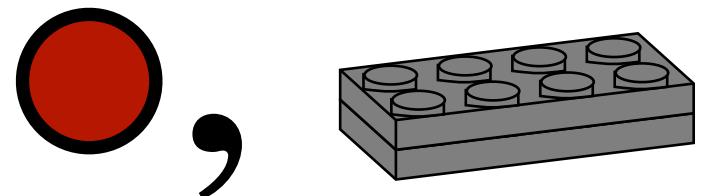


annihilating element?

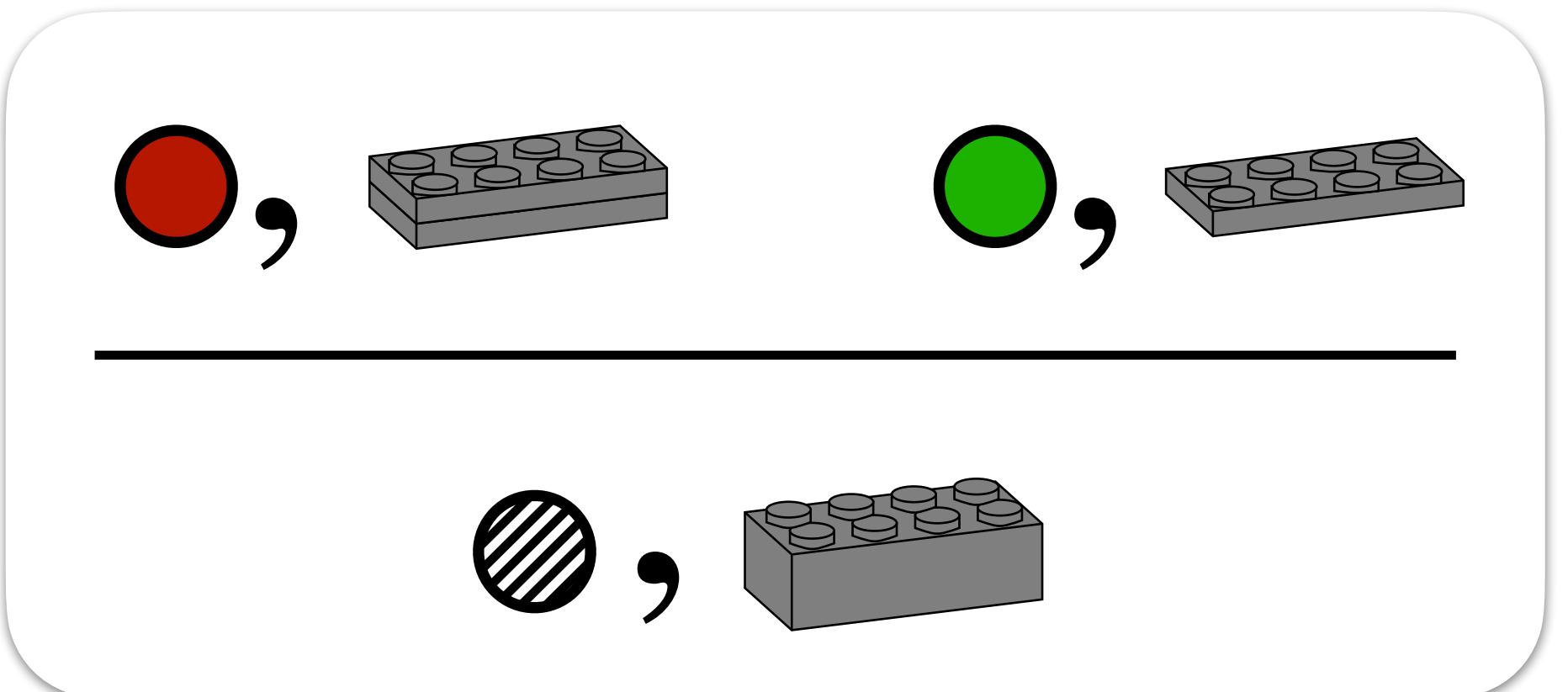
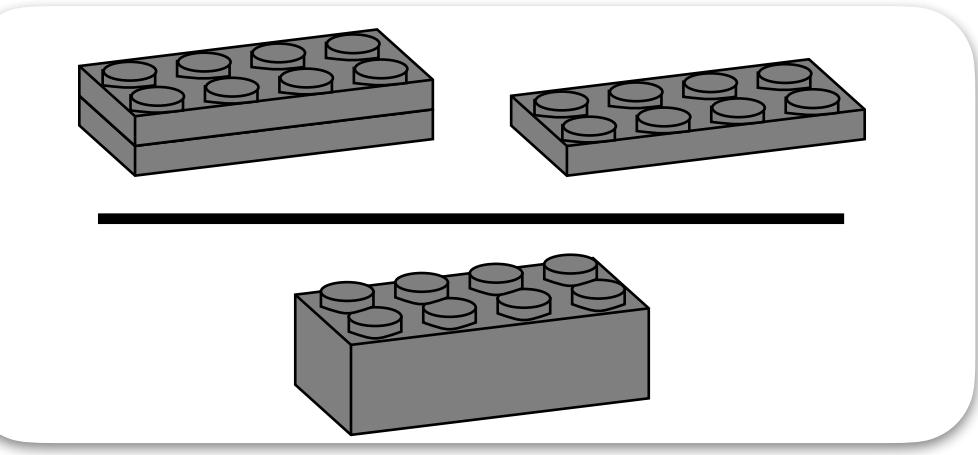
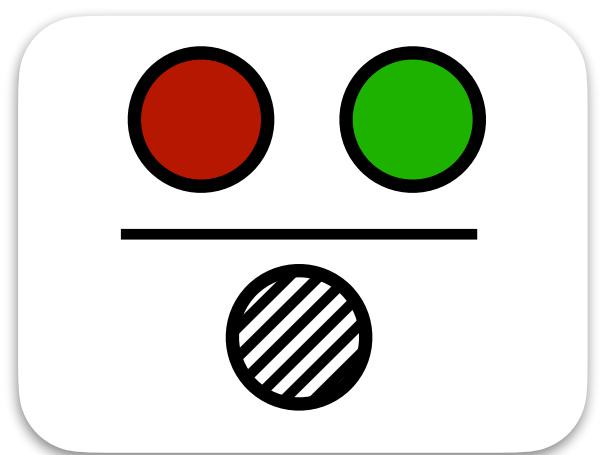


Composition of rules

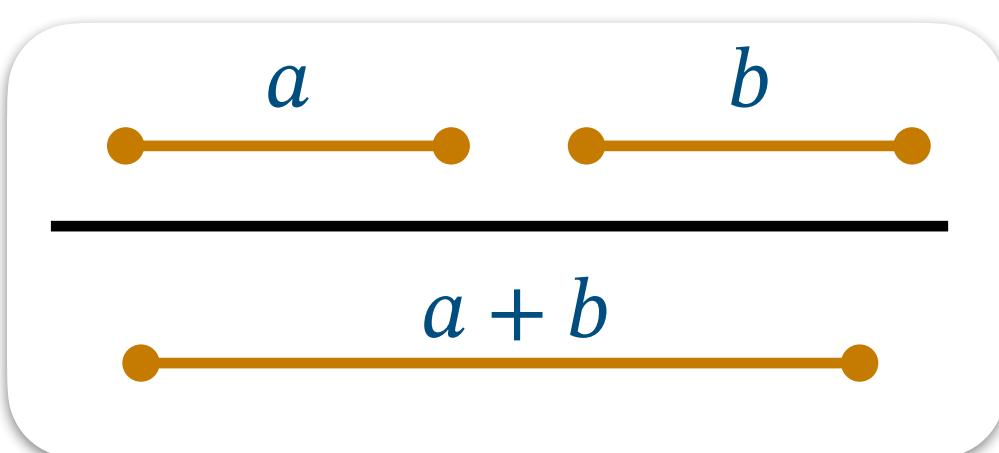
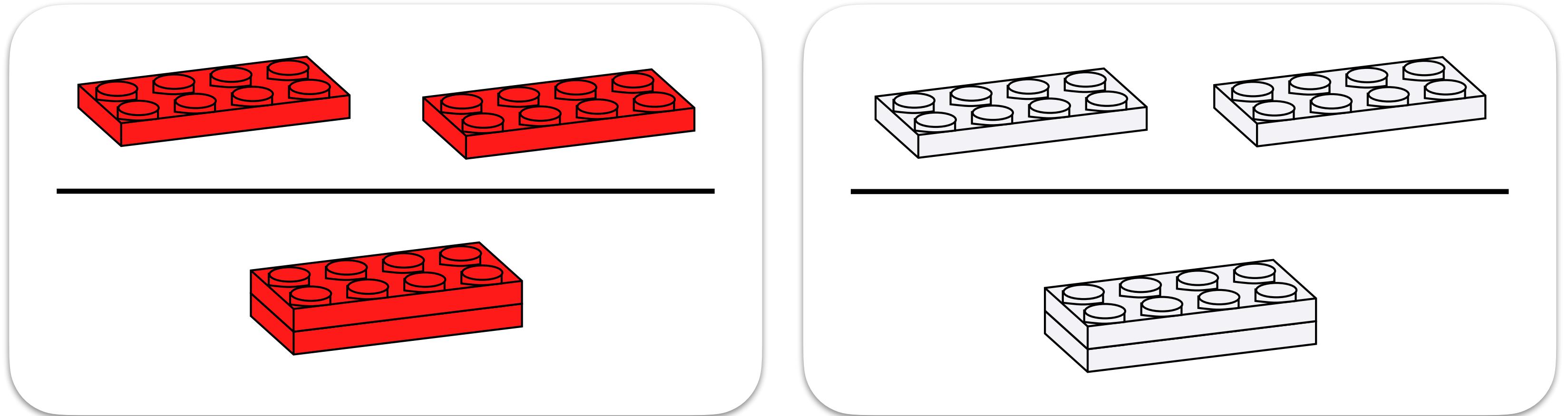
- Each brick has a color and a shape.



- Composing component-wise:

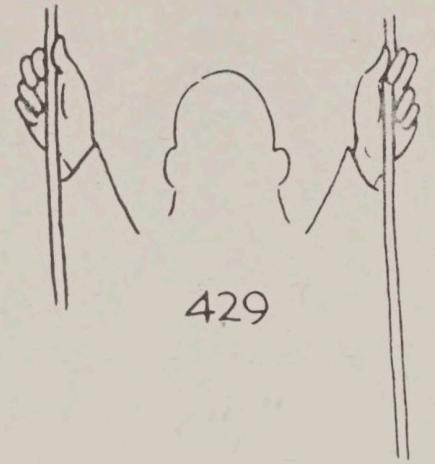


Isomorphisms

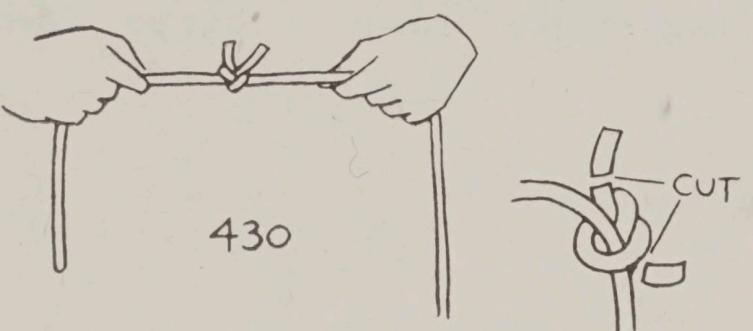


Rope tricks

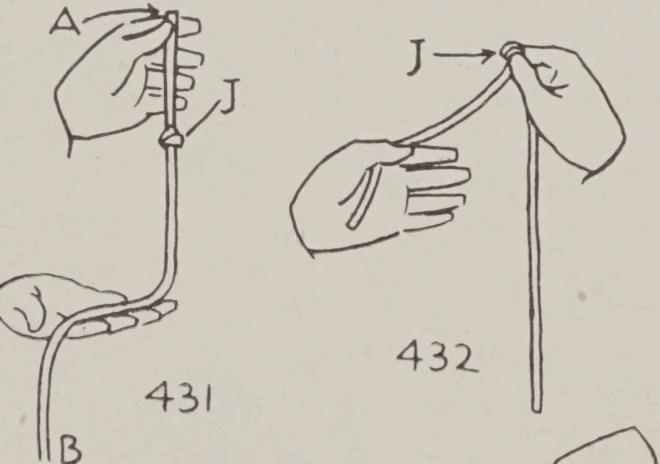
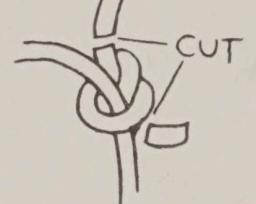
ENCYCLOPEDIA OF ROPE TRICKS



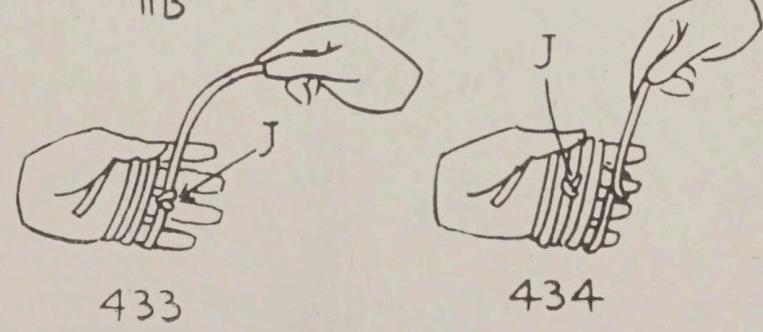
429



430



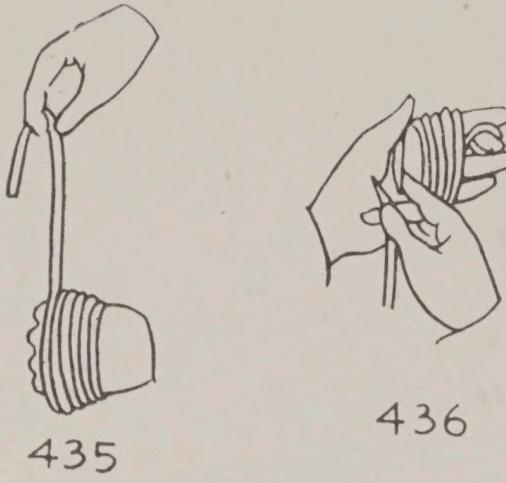
431



432

482

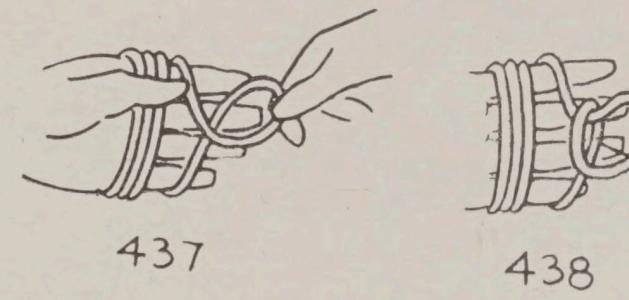
ENCYCLOPEDIA OF ROPE TRICKS



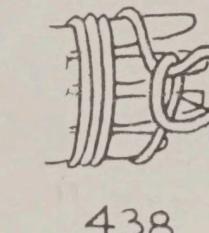
435



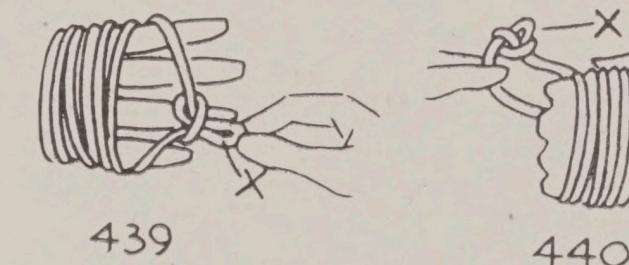
436



437



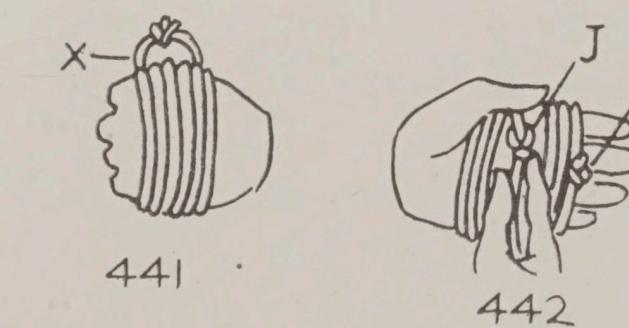
438



439



440



441

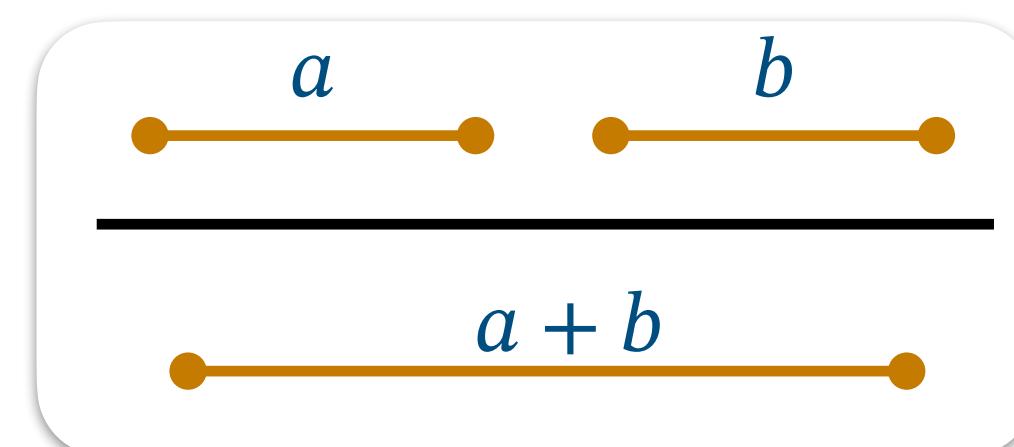
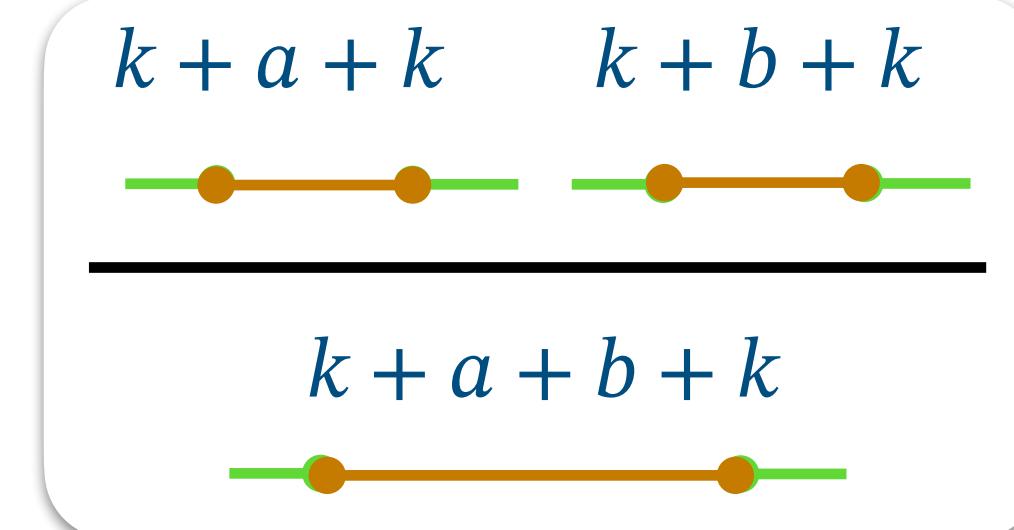
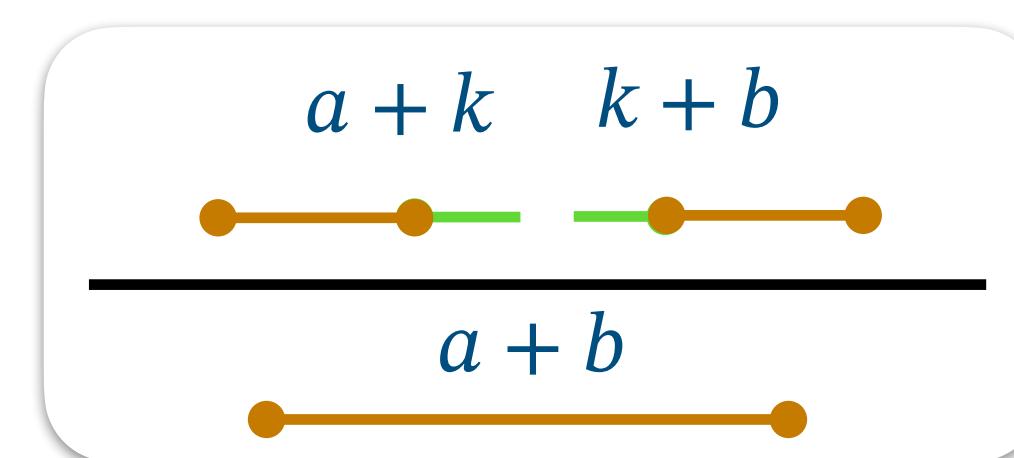
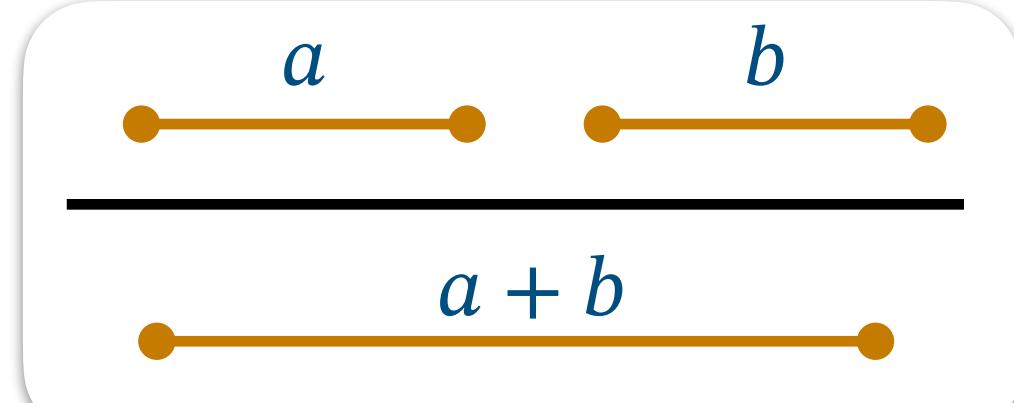
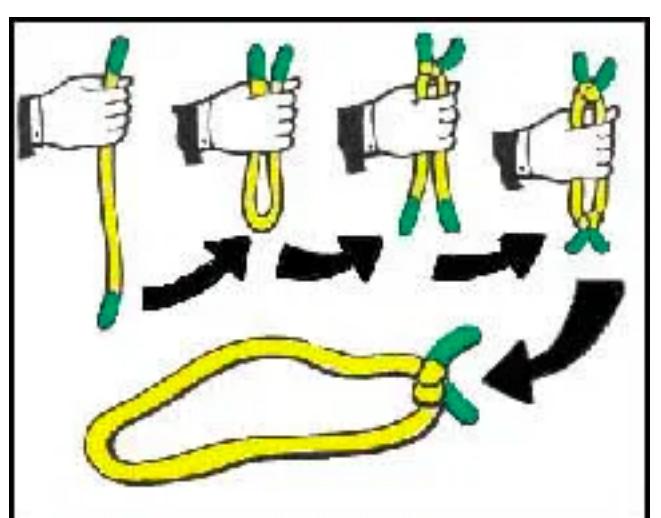


442

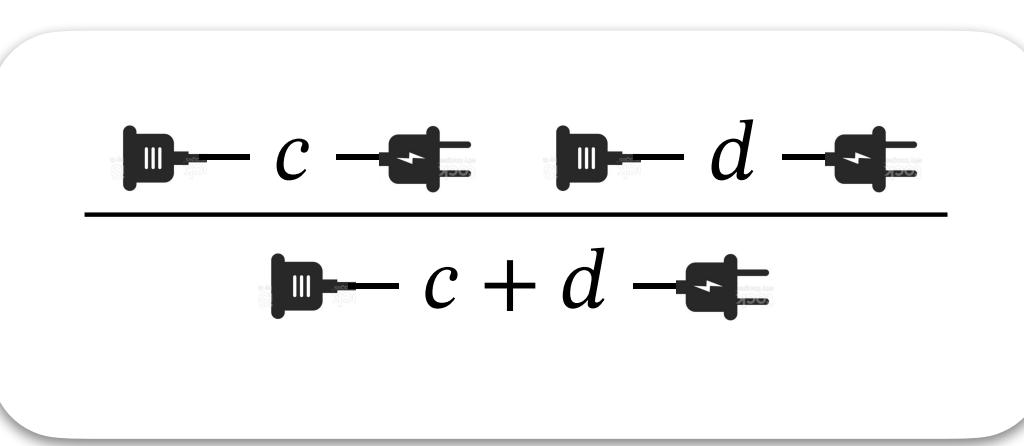
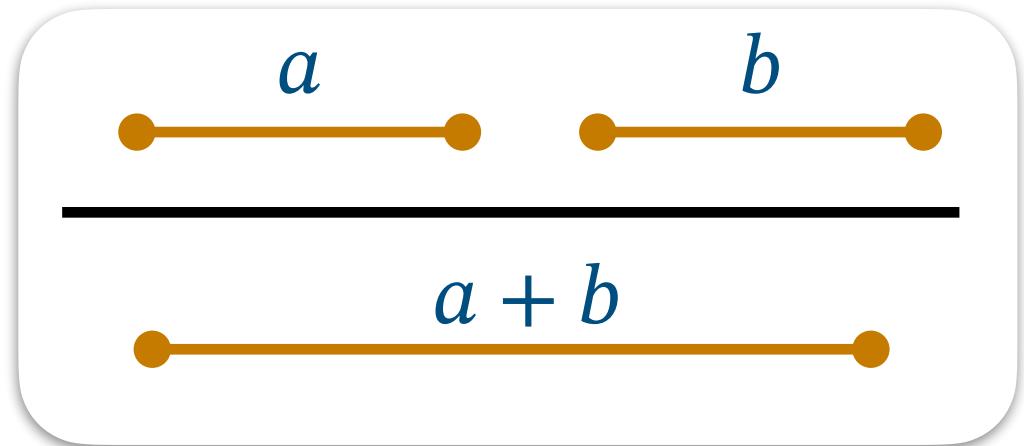
483

Rope tricks

account for knot material



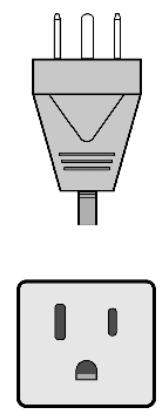
Isomorphisms?



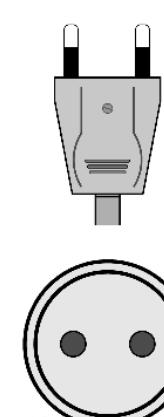
What about the interfaces?



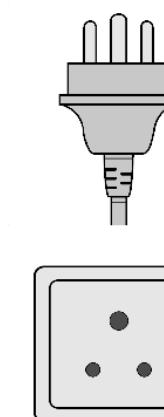
(a) Type A.



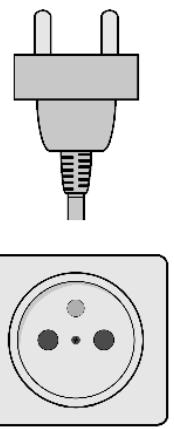
(b) Type B.



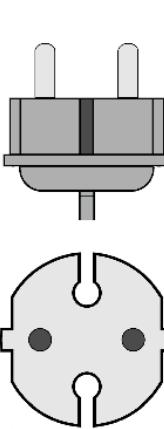
(c) Type C.



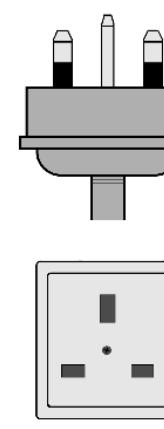
(d) Type D.



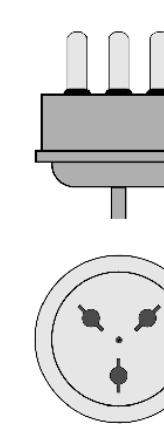
(e) Type E.



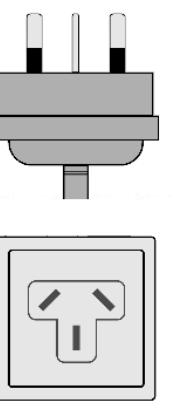
(f) Type F.



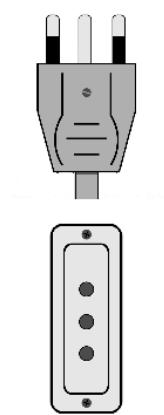
(g) Type G.



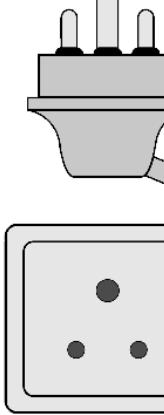
(h) Type H.



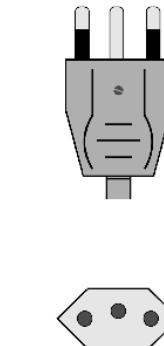
(i) Type I.



(j) Type L.



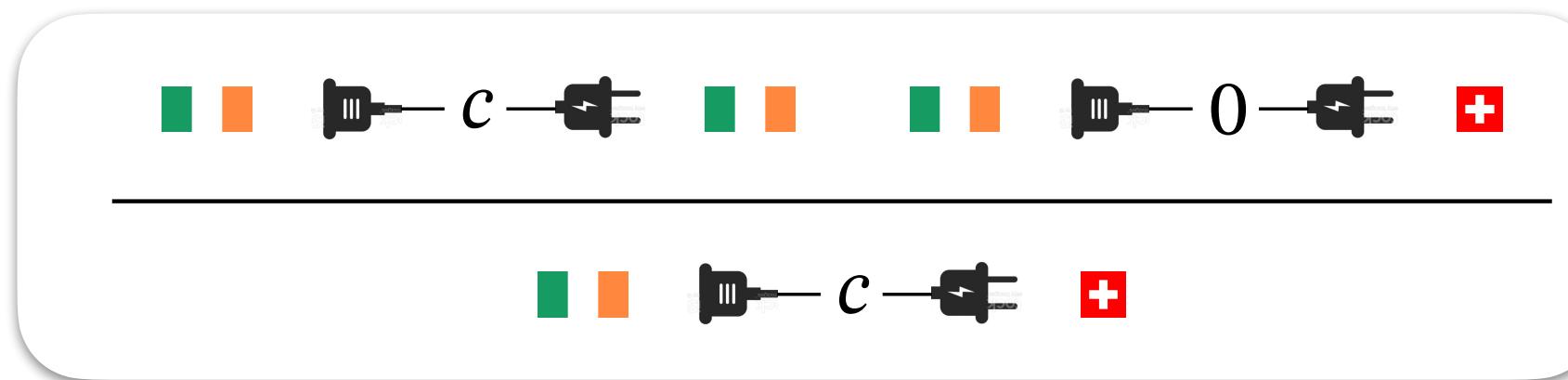
(k) Type M.



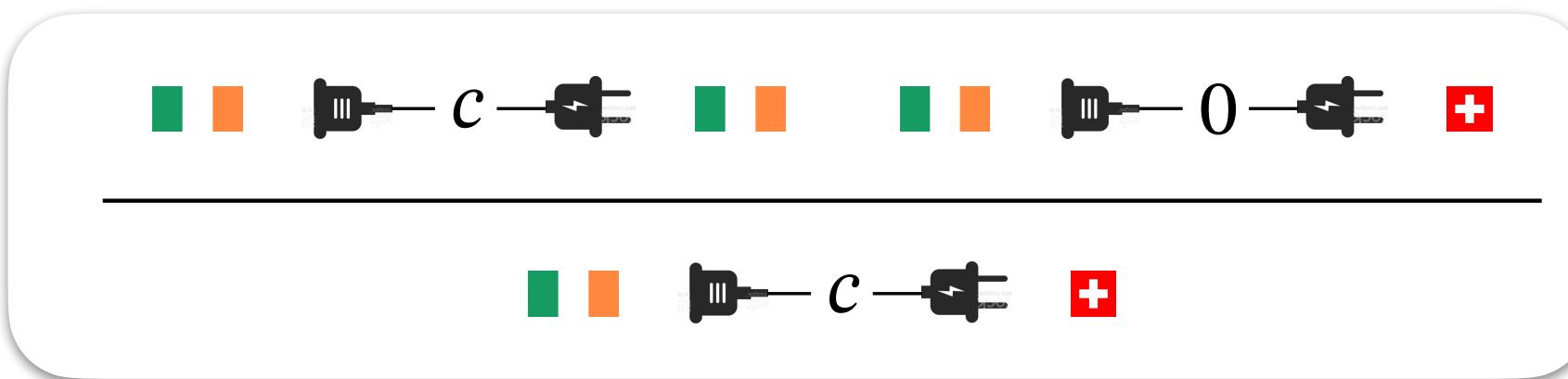
(l) Type N.



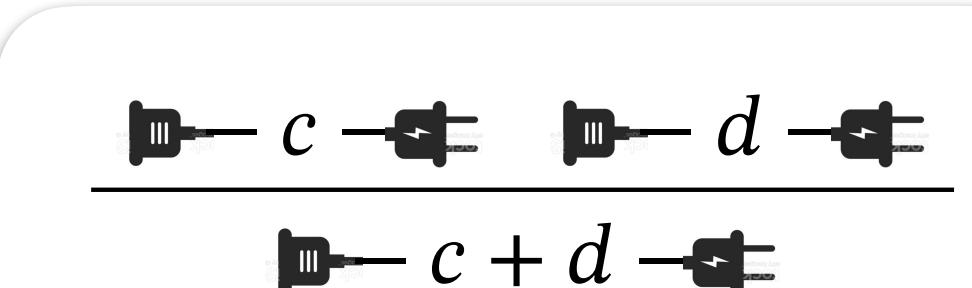
Cords with interfaces



Cords with interfaces



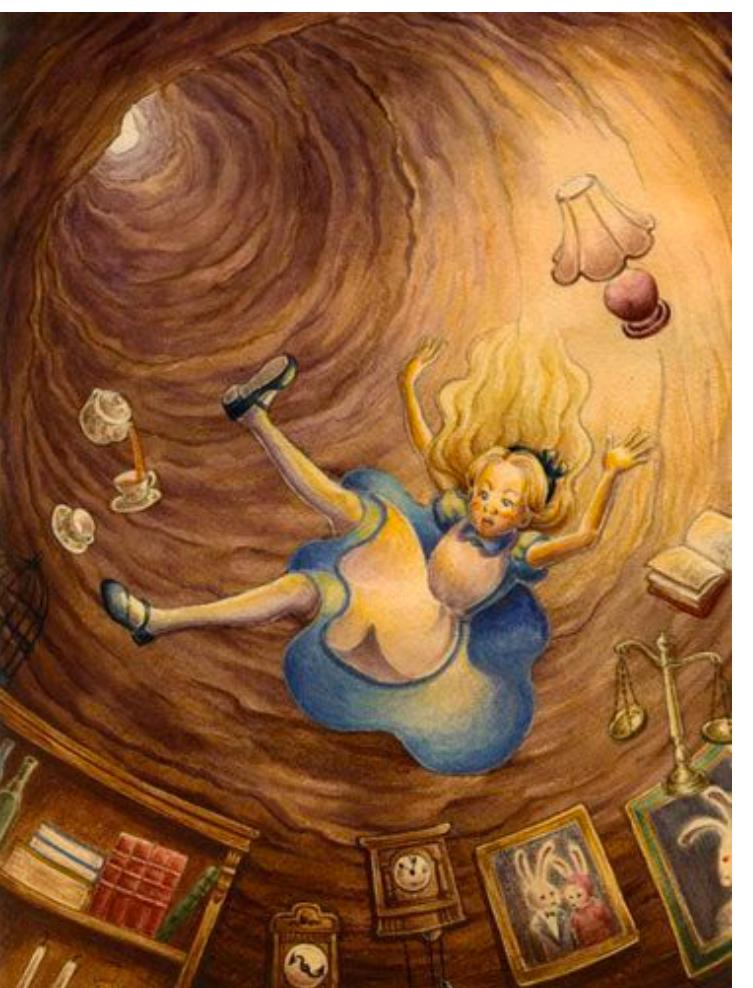
forget interfaces



Summary

- ▶ **Composition** creates complex structures from small.
- ▶ There are **many ways to compose** the same objects.
- ▶ **We look for what is common** among different composition structures.

- ▶ Composition is about **interfaces**.
- ▶ **Up the ladder of abstraction:** The rules themselves can be composed.
- ▶ ...



algebra



*category
theory*

